# Intersection numbers of integral standard generalized table algebras with non-cyclotomic minimal splitting fields

Akihide Hanaki

November 27, 2016

In [1], it was asked whether minimal splitting fields of adjacency algebras of (commutative) association schemes are abelian number fields or not. This is still an open question. In [3], the author talked about necessary conditions for number fields to be non-abelian minimal splitting fields for association schemes of prime order. In [4], Toru Komatsu constructed possible number fields. For fields constructed by Komatsu, Eiichi Bannai computed possible intersection numbers in [5]. After that, Sho Teranishi computed many examples of intersection numbers in his master's thesis at Shinshu University [6].

Unfortunately, some parts of this story were published only in Japanese. In this note, we only give characteristic polynomials and intersection numbers obtained by Teranishi. Theoretical parts of his method are due to [4, 5], and computational parts are basic, complicated, and not proved theoretically. We will append his program for GAP [2] with package ALNUTH.

## 1 Case $d = 4$

### 1.1 $d = 4$, $p = 2713$

$$\begin{aligned} f_1(X) &= X^4 + X^3 - 1017X^2 - 9665X + 60608 \\ f_2(X) &= X^4 + X^3 - 1017X^2 - 1526X + 215249 \\ f_3(X) &= X^4 + X^3 - 1017X^2 - 1526X + 231527 \\ f_4(X) &= X^4 + X^3 - 1017X^2 + 14752X - 45199 \end{aligned}$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 678 & 179 & 162 & 160 & 176 \\ 0 & 162 & 164 & 185 & 167 \\ 0 & 160 & 185 & 166 & 167 \\ 0 & 176 & 167 & 167 & 168 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 162 & 164 & 185 & 167 \\ 678 & 164 & 170 & 168 & 175 \\ 0 & 185 & 168 & 167 & 158 \\ 0 & 167 & 175 & 158 & 178 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 160 & 185 & 166 & 167 \\ 0 & 185 & 168 & 167 & 158 \\ 678 & 166 & 167 & 170 & 174 \\ 0 & 167 & 158 & 174 & 179 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 176 & 167 & 167 & 168 \\ 0 & 167 & 175 & 158 & 178 \\ 0 & 167 & 158 & 174 & 179 \\ 678 & 168 & 178 & 179 & 152 \end{pmatrix}$$

### 1.2 $d = 4,\ p = 2777$

$$
\begin{aligned}
f_1(X) &= X^4 + X^3 - 1041X^2 + 1215X + 229352\\
f_2(X) &= X^4 + X^3 - 1041X^2 - 7116X - 12247\\
f_3(X) &= X^4 + X^3 - 1041X^2 + 9546X + 37739\\
f_4(X) &= X^4 + X^3 - 1041X^2 + 9546X + 54401
\end{aligned}
$$

$$
B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0\\ 694 & 180 & 156 & 179 & 178\\ 0 & 156 & 180 & 179 & 179\\ 0 & 179 & 179 & 166 & 170\\ 0 & 178 & 179 & 170 & 167 \end{pmatrix}, \quad
B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0\\ 0 & 156 & 180 & 179 & 179\\ 694 & 180 & 171 & 172 & 170\\ 0 & 179 & 172 & 182 & 161\\ 0 & 179 & 170 & 161 & 184 \end{pmatrix},
$$

$$
B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0\\ 0 & 179 & 179 & 166 & 170\\ 0 & 179 & 172 & 182 & 161\\ 694 & 166 & 182 & 162 & 183\\ 0 & 170 & 161 & 183 & 180 \end{pmatrix}, \quad
B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1\\ 0 & 178 & 179 & 170 & 167\\ 0 & 179 & 170 & 161 & 184\\ 0 & 170 & 161 & 183 & 180\\ 694 & 167 & 184 & 180 & 162 \end{pmatrix}
$$

$$
\begin{aligned}
f_5(X) &= X^4 + X^3 - 1041X^2 - 1562X + 93279\\
f_6(X) &= X^4 + X^3 - 1041X^2 + 1215X + 51624\\
f_7(X) &= X^4 + X^3 - 1041X^2 + 3992X + 171035\\
f_8(X) &= X^4 + X^3 - 1041X^2 + 15100X - 56679
\end{aligned}
$$

$$
B_5 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0\\ 694 & 174 & 180 & 160 & 179\\ 0 & 180 & 160 & 183 & 171\\ 0 & 160 & 183 & 183 & 168\\ 0 & 179 & 171 & 168 & 176 \end{pmatrix}, \quad
B_6 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0\\ 0 & 180 & 160 & 183 & 171\\ 694 & 160 & 171 & 178 & 184\\ 0 & 183 & 178 & 168 & 165\\ 0 & 171 & 184 & 165 & 174 \end{pmatrix},
$$

$$
B_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0\\ 0 & 160 & 183 & 183 & 168\\ 0 & 183 & 178 & 168 & 165\\ 694 & 183 & 168 & 168 & 174\\ 0 & 168 & 165 & 174 & 187 \end{pmatrix}, \quad
B_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1\\ 0 & 179 & 171 & 168 & 176\\ 0 & 171 & 184 & 165 & 174\\ 0 & 168 & 165 & 174 & 187\\ 694 & 176 & 174 & 187 & 156 \end{pmatrix}
$$

### 1.3 $d = 4,\ p = 2857$

$$\begin{aligned} f_1(X) &= X^4 + X^3 - 1071X^2 - 7321X - 8850 \\ f_2(X) &= X^4 + X^3 - 1071X^2 - 4464X + 102573 \\ f_3(X) &= X^4 + X^3 - 1071X^2 + 1250X - 279 \\ f_4(X) &= X^4 + X^3 - 1071X^2 + 1250X + 85431 \end{aligned}$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 714 & 185 & 162 & 188 & 178 \\ 0 & 162 & 186 & 183 & 183 \\ 0 & 188 & 183 & 166 & 177 \\ 0 & 178 & 183 & 177 & 176 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 162 & 186 & 183 & 183 \\ 714 & 186 & 182 & 180 & 165 \\ 0 & 183 & 180 & 177 & 174 \\ 0 & 183 & 165 & 174 & 192 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 188 & 183 & 166 & 177 \\ 0 & 183 & 180 & 177 & 174 \\ 714 & 166 & 177 & 176 & 194 \\ 0 & 177 & 174 & 194 & 169 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 178 & 183 & 177 & 176 \\ 0 & 183 & 165 & 174 & 192 \\ 0 & 177 & 174 & 194 & 169 \\ 714 & 176 & 192 & 169 & 176 \end{pmatrix}$$

### 1.4 $d = 4,\ p = 3137$

$$\begin{aligned} f_1(X) &= X^4 + X^3 - 1176X^2 - 9607X + 19594 \\ f_2(X) &= X^4 + X^3 - 1176X^2 - 9607X + 113704 \\ f_3(X) &= X^4 + X^3 - 1176X^2 - 196X + 264280 \\ f_4(X) &= X^4 + X^3 - 1176X^2 + 9215X - 18050 \end{aligned}$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 784 & 204 & 181 & 192 & 206 \\ 0 & 181 & 196 & 208 & 199 \\ 0 & 192 & 208 & 194 & 190 \\ 0 & 206 & 199 & 190 & 189 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 181 & 196 & 208 & 199 \\ 784 & 196 & 204 & 182 & 201 \\ 0 & 208 & 182 & 204 & 190 \\ 0 & 199 & 201 & 190 & 194 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 192 & 208 & 194 & 190 \\ 0 & 208 & 182 & 204 & 190 \\ 784 & 194 & 204 & 195 & 190 \\ 0 & 190 & 190 & 190 & 214 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 206 & 199 & 190 & 189 \\ 0 & 199 & 201 & 190 & 194 \\ 0 & 190 & 190 & 190 & 214 \\ 784 & 189 & 194 & 214 & 186 \end{pmatrix}$$

## 1.5 $d = 4,\ p = 4409$

$$f_1(X) = X^4 + X^3 - 1653X^2 - 28934X - 61881$$
$$f_2(X) = X^4 + X^3 - 1653X^2 - 6889X + 312884$$
$$f_3(X) = X^4 + X^3 - 1653X^2 - 2480X + 625923$$
$$f_4(X) = X^4 + X^3 - 1653X^2 + 15156X + 70389$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1102 & 294 & 268 & 260 & 279 \\ 0 & 268 & 264 & 291 & 279 \\ 0 & 260 & 291 & 275 & 276 \\ 0 & 279 & 279 & 276 & 268 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 268 & 264 & 291 & 279 \\ 1102 & 264 & 279 & 270 & 288 \\ 0 & 291 & 270 & 280 & 261 \\ 0 & 279 & 288 & 261 & 274 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 260 & 291 & 275 & 276 \\ 0 & 291 & 270 & 280 & 261 \\ 1102 & 275 & 280 & 276 & 270 \\ 0 & 276 & 261 & 270 & 295 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 279 & 279 & 276 & 268 \\ 0 & 279 & 288 & 261 & 274 \\ 0 & 276 & 261 & 270 & 295 \\ 1102 & 268 & 274 & 295 & 264 \end{pmatrix}$$

## 1.6 $d = 4,\ p = 5081$

$$f_1(X) = X^4 + X^3 - 1905X^2 - 43506X - 271377$$
$$f_2(X) = X^4 + X^3 - 1905X^2 - 38425X - 144352$$
$$f_3(X) = X^4 + X^3 - 1905X^2 - 33344X - 149433$$
$$f_4(X) = X^4 + X^3 - 1905X^2 - 33344X - 57975$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1270 & 342 & 304 & 316 & 307 \\ 0 & 304 & 312 & 327 & 327 \\ 0 & 316 & 327 & 303 & 324 \\ 0 & 307 & 327 & 324 & 312 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 304 & 312 & 327 & 327 \\ 1270 & 312 & 339 & 318 & 300 \\ 0 & 327 & 318 & 304 & 321 \\ 0 & 327 & 300 & 321 & 322 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 316 & 327 & 303 & 324 \\ 0 & 327 & 318 & 304 & 321 \\ 1270 & 303 & 304 & 336 & 326 \\ 0 & 324 & 321 & 326 & 299 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 307 & 327 & 324 & 312 \\ 0 & 327 & 300 & 321 & 322 \\ 0 & 324 & 321 & 326 & 299 \\ 1270 & 312 & 322 & 299 & 336 \end{pmatrix}$$

## 1.7 $d = 4,\ p = 5281$

$$f_1(X) = X^4 + X^3 - 1980X^2 + 15513X - 28406$$
$$f_2(X) = X^4 + X^3 - 1980X^2 + 15513X + 193396$$
$$f_3(X) = X^4 + X^3 - 1980X^2 + 26075X + 119462$$
$$f_4(X) = X^4 + X^3 - 1980X^2 + 31356X - 123464$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1320 & 320 & 313 & 338 & 348 \\ 0 & 313 & 348 & 331 & 328 \\ 0 & 338 & 331 & 329 & 322 \\ 0 & 348 & 328 & 322 & 322 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 313 & 348 & 331 & 328 \\ 1320 & 348 & 320 & 317 & 334 \\ 0 & 331 & 317 & 350 & 322 \\ 0 & 328 & 334 & 322 & 336 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 338 & 331 & 329 & 322 \\ 0 & 331 & 317 & 350 & 322 \\ 1320 & 329 & 350 & 314 & 326 \\ 0 & 322 & 322 & 326 & 350 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 348 & 328 & 322 & 322 \\ 0 & 328 & 334 & 322 & 336 \\ 0 & 322 & 322 & 326 & 350 \\ 1320 & 322 & 336 & 350 & 311 \end{pmatrix}$$

## 1.8 $d = 4,\ p = 5297$

$$f_1(X) = X^4 + X^3 - 1986X^2 - 331X + 30106$$
$$f_2(X) = X^4 + X^3 - 1986X^2 - 331X + 379708$$
$$f_3(X) = X^4 + X^3 - 1986X^2 + 15560X + 443272$$
$$f_4(X) = X^4 + X^3 - 1986X^2 + 31451X - 128804$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1324 & 330 & 310 & 336 & 347 \\ 0 & 310 & 343 & 340 & 331 \\ 0 & 336 & 340 & 326 & 322 \\ 0 & 347 & 331 & 322 & 324 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 310 & 343 & 340 & 331 \\ 1324 & 343 & 330 & 314 & 336 \\ 0 & 340 & 314 & 348 & 322 \\ 0 & 331 & 336 & 322 & 335 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 336 & 340 & 326 & 322 \\ 0 & 340 & 314 & 348 & 322 \\ 1324 & 326 & 348 & 321 & 328 \\ 0 & 322 & 322 & 328 & 352 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 347 & 331 & 322 & 324 \\ 0 & 331 & 336 & 322 & 335 \\ 0 & 322 & 322 & 328 & 352 \\ 1324 & 324 & 335 & 352 & 312 \end{pmatrix}$$

### 1.9 $d = 4,\ p = 7057$

$$f_1(X) = X^4 + X^3 - 2646X^2 - 28669X + 617901$$
$$f_2(X) = X^4 + X^3 - 2646X^2 - 28669X + 787269$$
$$f_3(X) = X^4 + X^3 - 2646X^2 - 441X + 1718793$$
$$f_4(X) = X^4 + X^3 - 2646X^2 + 56015X - 313623$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1764 & 452 & 425 & 432 & 454 \\ 0 & 425 & 433 & 465 & 441 \\ 0 & 432 & 465 & 438 & 429 \\ 0 & 454 & 441 & 429 & 440 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 425 & 433 & 465 & 441 \\ 1764 & 433 & 452 & 426 & 452 \\ 0 & 465 & 426 & 444 & 429 \\ 0 & 441 & 452 & 429 & 442 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 432 & 465 & 438 & 429 \\ 0 & 465 & 426 & 444 & 429 \\ 1764 & 438 & 444 & 440 & 441 \\ 0 & 429 & 429 & 441 & 465 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 454 & 441 & 429 & 440 \\ 0 & 441 & 452 & 429 & 442 \\ 0 & 429 & 429 & 441 & 465 \\ 1764 & 440 & 442 & 465 & 416 \end{pmatrix}$$

### 1.10 $d = 4,\ p = 7481$

$$f_1(X) = X^4 + X^3 - 2805X^2 - 71537X - 396756$$
$$f_2(X) = X^4 + X^3 - 2805X^2 - 49094X + 29661$$
$$f_3(X) = X^4 + X^3 - 2805X^2 - 34132X + 777761$$
$$f_4(X) = X^4 + X^3 - 2805X^2 - 19170X + 74547$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1870 & 495 & 458 & 448 & 468 \\ 0 & 458 & 452 & 483 & 477 \\ 0 & 448 & 483 & 468 & 471 \\ 0 & 468 & 477 & 471 & 454 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 458 & 452 & 483 & 477 \\ 1870 & 452 & 486 & 451 & 480 \\ 0 & 483 & 451 & 474 & 462 \\ 0 & 477 & 480 & 462 & 451 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 448 & 483 & 468 & 471 \\ 0 & 483 & 451 & 474 & 462 \\ 1870 & 468 & 474 & 480 & 447 \\ 0 & 471 & 462 & 447 & 490 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 468 & 477 & 471 & 454 \\ 0 & 477 & 480 & 462 & 451 \\ 0 & 471 & 462 & 447 & 490 \\ 1870 & 454 & 451 & 490 & 474 \end{pmatrix}$$

### 1.11 $d = 4$, $p = 7753$

$$f_1(X) = X^4 + X^3 - 2907X^2 - 74138X - 441709$$
$$f_2(X) = X^4 + X^3 - 2907X^2 - 27620X + 256061$$
$$f_3(X) = X^4 + X^3 - 2907X^2 - 27620X + 1139903$$
$$f_4(X) = X^4 + X^3 - 2907X^2 - 4361X + 418874$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1938 & 512 & 481 & 462 & 482 \\ 0 & 481 & 466 & 500 & 491 \\ 0 & 462 & 500 & 485 & 491 \\ 0 & 482 & 491 & 491 & 474 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 481 & 466 & 500 & 491 \\ 1938 & 466 & 494 & 473 & 504 \\ 0 & 500 & 473 & 492 & 473 \\ 0 & 491 & 504 & 473 & 470 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 462 & 500 & 485 & 491 \\ 0 & 500 & 473 & 492 & 473 \\ 1938 & 485 & 492 & 494 & 466 \\ 0 & 491 & 473 & 466 & 508 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 482 & 491 & 491 & 474 \\ 0 & 491 & 504 & 473 & 470 \\ 0 & 491 & 473 & 466 & 508 \\ 1938 & 474 & 470 & 508 & 485 \end{pmatrix}$$

### 1.12 $d = 4$, $p = 7873$

$$f_1(X) = X^4 + X^3 - 2952X^2 - 492X + 179080$$
$$f_2(X) = X^4 + X^3 - 2952X^2 + 23127X + 1265554$$
$$f_3(X) = X^4 + X^3 - 2952X^2 + 38873X + 257810$$
$$f_4(X) = X^4 + X^3 - 2952X^2 + 70365X - 293300$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1968 & 491 & 466 & 506 & 504 \\ 0 & 466 & 510 & 502 & 490 \\ 0 & 506 & 502 & 476 & 484 \\ 0 & 504 & 490 & 484 & 490 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 466 & 510 & 502 & 490 \\ 1968 & 510 & 482 & 485 & 490 \\ 0 & 502 & 485 & 506 & 475 \\ 0 & 490 & 490 & 475 & 513 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 506 & 502 & 476 & 484 \\ 0 & 502 & 485 & 506 & 475 \\ 1968 & 476 & 506 & 476 & 509 \\ 0 & 484 & 475 & 509 & 500 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 504 & 490 & 484 & 490 \\ 0 & 490 & 490 & 475 & 513 \\ 0 & 484 & 475 & 509 & 500 \\ 1968 & 490 & 513 & 500 & 464 \end{pmatrix}$$

## 1.13 $d = 4,\ p = 8017$

$$
\begin{aligned}
f_1(X) &= X^4 + X^3 - 3006X^2 - 16535X + 772106\\
f_2(X) &= X^4 + X^3 - 3006X^2 - 16535X + 1253126\\
f_3(X) &= X^4 + X^3 - 3006X^2 + 7516X + 1782248\\
f_4(X) &= X^4 + X^3 - 3006X^2 + 79669X - 574750
\end{aligned}
$$

$$
B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0\\ 2004 & 506 & 479 & 502 & 516\\ 0 & 479 & 504 & 524 & 497\\ 0 & 502 & 524 & 490 & 488\\ 0 & 516 & 497 & 488 & 503 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0\\ 0 & 479 & 504 & 524 & 497\\ 2004 & 504 & 506 & 482 & 511\\ 0 & 524 & 482 & 510 & 488\\ 0 & 497 & 511 & 488 & 508 \end{pmatrix},
$$

$$
B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0\\ 0 & 502 & 524 & 490 & 488\\ 0 & 524 & 482 & 510 & 488\\ 2004 & 490 & 510 & 497 & 506\\ 0 & 488 & 488 & 506 & 522 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1\\ 0 & 516 & 497 & 488 & 503\\ 0 & 497 & 511 & 488 & 508\\ 0 & 488 & 488 & 506 & 522\\ 2004 & 503 & 508 & 522 & 470 \end{pmatrix}
$$

## 1.14 $d = 4,\ p = 8713$

$$
\begin{aligned}
f_1(X) &= X^4 + X^3 - 3267X^2 - 48466X - 141348\\
f_2(X) &= X^4 + X^3 - 3267X^2 + 3812X + 2367996\\
f_3(X) &= X^4 + X^3 - 3267X^2 + 38664X + 590544\\
f_4(X) &= X^4 + X^3 - 3267X^2 + 56090X + 485988
\end{aligned}
$$

$$
B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0\\ 2178 & 560 & 514 & 552 & 551\\ 0 & 514 & 554 & 558 & 552\\ 0 & 552 & 558 & 528 & 540\\ 0 & 551 & 552 & 540 & 535 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0\\ 0 & 514 & 554 & 558 & 552\\ 2178 & 554 & 542 & 543 & 538\\ 0 & 558 & 543 & 555 & 522\\ 0 & 552 & 538 & 522 & 566 \end{pmatrix},
$$

$$
B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0\\ 0 & 552 & 558 & 528 & 540\\ 0 & 558 & 543 & 555 & 522\\ 2178 & 528 & 555 & 530 & 564\\ 0 & 540 & 522 & 564 & 552 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1\\ 0 & 551 & 552 & 540 & 535\\ 0 & 552 & 538 & 522 & 566\\ 0 & 540 & 522 & 564 & 552\\ 2178 & 535 & 566 & 552 & 524 \end{pmatrix}
$$

## 1.15 $d = 4,\ p = 8761$

$$\begin{aligned} f_1(X) &= X^4 + X^3 - 3285X^2 - 75016X - 166767 \\ f_2(X) &= X^4 + X^3 - 3285X^2 - 22450X + 621723 \\ f_3(X) &= X^4 + X^3 - 3285X^2 - 13689X + 2172420 \\ f_4(X) &= X^4 + X^3 - 3285X^2 + 30116X + 306327 \end{aligned}$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 2190 & 572 & 544 & 522 & 551 \\ 0 & 544 & 527 & 567 & 552 \\ 0 & 522 & 567 & 552 & 549 \\ 0 & 551 & 552 & 549 & 538 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 544 & 527 & 567 & 552 \\ 2190 & 527 & 554 & 540 & 568 \\ 0 & 567 & 540 & 552 & 531 \\ 0 & 552 & 568 & 531 & 539 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 522 & 567 & 552 & 549 \\ 0 & 567 & 540 & 552 & 531 \\ 2190 & 552 & 552 & 551 & 534 \\ 0 & 549 & 531 & 534 & 576 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 551 & 552 & 549 & 538 \\ 0 & 552 & 568 & 531 & 539 \\ 0 & 549 & 531 & 534 & 576 \\ 2190 & 538 & 539 & 576 & 536 \end{pmatrix}$$

## 1.16 $d = 4,\ p = 9833$

$$\begin{aligned} f_1(X) &= X^4 + X^3 - 3687X^2 - 113694X - 952303 \\ f_2(X) &= X^4 + X^3 - 3687X^2 - 25197X + 2204090 \\ f_3(X) &= X^4 + X^3 - 3687X^2 + 4302X + 935633 \\ f_4(X) &= X^4 + X^3 - 3687X^2 + 4302X + 1761605 \end{aligned}$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 2458 & 648 & 592 & 612 & 605 \\ 0 & 592 & 608 & 629 & 629 \\ 0 & 612 & 629 & 597 & 620 \\ 0 & 605 & 629 & 620 & 604 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 592 & 608 & 629 & 629 \\ 2458 & 608 & 621 & 628 & 600 \\ 0 & 629 & 628 & 608 & 593 \\ 0 & 629 & 600 & 593 & 636 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 612 & 629 & 597 & 620 \\ 0 & 629 & 628 & 608 & 593 \\ 2458 & 597 & 608 & 612 & 640 \\ 0 & 620 & 593 & 640 & 605 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 605 & 629 & 620 & 604 \\ 0 & 629 & 600 & 593 & 636 \\ 0 & 620 & 593 & 640 & 605 \\ 2458 & 604 & 636 & 605 & 612 \end{pmatrix}$$

# 2 Case $d = 5$

## 2.1 $d = 5$, $p = 1951$

$$f_1(X) = X^5 + X^4 - 780X^3 - 156X^2 + 147480X - 109600$$
$$f_2(X) = X^5 + X^4 - 780X^3 + 1795X^2 + 3106X - 344$$
$$f_3(X) = X^5 + X^4 - 780X^3 + 7648X^2 + 7008X - 140816$$
$$f_4(X) = X^5 + X^4 - 780X^3 - 9911X^2 - 24208X - 15952$$
$$f_5(X) = X^5 + X^4 - 780X^3 - 6009X^2 + 46028X + 327424$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 390 & 77 & 74 & 74 & 80 & 84 \\ 0 & 74 & 86 & 72 & 78 & 80 \\ 0 & 74 & 72 & 92 & 80 & 72 \\ 0 & 80 & 78 & 80 & 66 & 86 \\ 0 & 84 & 80 & 72 & 86 & 68 \end{pmatrix}, B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 74 & 86 & 72 & 78 & 80 \\ 390 & 86 & 74 & 90 & 71 & 68 \\ 0 & 72 & 90 & 76 & 76 & 76 \\ 0 & 78 & 71 & 76 & 82 & 83 \\ 0 & 80 & 68 & 76 & 83 & 83 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 74 & 72 & 92 & 80 & 72 \\ 0 & 72 & 90 & 76 & 76 & 76 \\ 390 & 92 & 76 & 65 & 76 & 80 \\ 0 & 80 & 76 & 76 & 78 & 80 \\ 0 & 72 & 76 & 80 & 80 & 82 \end{pmatrix}, B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 80 & 78 & 80 & 66 & 86 \\ 0 & 78 & 71 & 76 & 82 & 83 \\ 0 & 80 & 76 & 76 & 78 & 80 \\ 390 & 66 & 82 & 78 & 92 & 71 \\ 0 & 86 & 83 & 80 & 71 & 70 \end{pmatrix},$$

$$B_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 84 & 80 & 72 & 86 & 68 \\ 0 & 80 & 68 & 76 & 83 & 83 \\ 0 & 72 & 76 & 80 & 80 & 82 \\ 0 & 86 & 83 & 80 & 71 & 70 \\ 390 & 68 & 83 & 82 & 70 & 86 \end{pmatrix}$$

### 2.2 $d = 5,\ p = 2141$

$$f_0(X) = X^5 + X^4 - 856X^3 + 4539X^2 + 88449X - 66173$$

$$f_1(X) = X^5 + X^4 - 856X^3 - 8307X^2 + 88449X + 1017173$$
$$f_2(X) = X^5 + X^4 - 856X^3 - 4025X^2 + 28501X + 40877$$
$$f_3(X) = X^5 + X^4 - 856X^3 + 257X^2 + 144115X - 597141$$
$$f_4(X) = X^5 + X^4 - 856X^3 + 257X^2 + 174089X - 194633$$
$$f_5(X) = X^5 + X^4 - 856X^3 + 257X^2 + 182653X - 288837$$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 428 & 96 & 88 & 78 & 87 & 78 \\ 0 & 88 & 83 & 83 & 94 & 80 \\ 0 & 78 & 83 & 82 & 86 & 99 \\ 0 & 87 & 94 & 86 & 78 & 83 \\ 0 & 78 & 80 & 99 & 83 & 88 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 88 & 83 & 83 & 94 & 80 \\ 428 & 83 & 90 & 92 & 70 & 92 \\ 0 & 83 & 92 & 82 & 85 & 86 \\ 0 & 94 & 70 & 85 & 88 & 91 \\ 0 & 80 & 92 & 86 & 91 & 79 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 78 & 83 & 82 & 86 & 99 \\ 0 & 83 & 92 & 82 & 85 & 86 \\ 428 & 82 & 82 & 84 & 96 & 83 \\ 0 & 86 & 85 & 96 & 87 & 74 \\ 0 & 99 & 86 & 83 & 74 & 86 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 87 & 94 & 86 & 78 & 83 \\ 0 & 94 & 70 & 85 & 88 & 91 \\ 0 & 86 & 85 & 96 & 87 & 74 \\ 428 & 78 & 88 & 87 & 84 & 90 \\ 0 & 83 & 91 & 74 & 90 & 90 \end{pmatrix},$$

$$B_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 78 & 80 & 99 & 83 & 88 \\ 0 & 80 & 92 & 86 & 91 & 79 \\ 0 & 99 & 86 & 83 & 74 & 86 \\ 0 & 83 & 91 & 74 & 90 & 90 \\ 428 & 88 & 79 & 86 & 90 & 84 \end{pmatrix}$$

# Appendix

This is an example of executing Teranishi's program on GAP.

```
gap> Read("teranishi.gap");;
gap> pol := DefinitionPolynomial4(2857,100,100);
[ x^4+x^3-1071*x^2-18749*x-88846, x^4+x^3-1071*x^2-7321*x-8850,
  x^4+x^3-1071*x^2-4464*x+102573, x^4+x^3-1071*x^2+1250*x-279,
  x^4+x^3-1071*x^2+1250*x+85431 ]
gap> MatB([pol[2], pol[3], pol[4], pol[5]], 2857);
[ [ [ 1, 0, 0, 0, 0 ], [ 0, 1, 0, 0, 0 ], [ 0, 0, 1, 0, 0 ],
      [ 0, 0, 0, 1, 0 ], [ 0, 0, 0, 0, 1 ] ],
  [ [ 0, 1, 0, 0, 0 ], [ 714, 185, 162, 188, 178 ],
      [ 0, 162, 186, 183, 183 ], [ 0, 188, 183, 166, 177 ],
      [ 0, 178, 183, 177, 176 ] ],
  [ [ 0, 0, 1, 0, 0 ], [ 0, 162, 186, 183, 183 ],
      [ 714, 186, 182, 180, 165 ], [ 0, 183, 180, 177, 174 ],
      [ 0, 183, 165, 174, 192 ] ],
  [ [ 0, 0, 0, 1, 0 ], [ 0, 188, 183, 166, 177 ], [ 0, 183, 180, 177, 174 ],
      [ 714, 166, 177, 176, 194 ], [ 0, 177, 174, 194, 169 ] ],
  [ [ 0, 0, 0, 0, 1 ], [ 0, 178, 183, 177, 176 ], [ 0, 183, 165, 174, 192 ],
      [ 0, 177, 174, 194, 169 ], [ 714, 176, 192, 169, 176 ] ] ]
```

The following is the program list.

```
########################
#### tearanishi.gap ####
########################
LoadPackage("alnuth");
x := Indeterminate(Rationals, "x");

##########################################################
PolynomialOfL:=function(a)
    local A,B,C,D,T,i,j,b,x;

    if IsPrimeInt(a) then
        A:=List([1..300],i->i-150);
        B:=List([1..100],i->3*i);
        C:=[];
        D:=[];

        for i in [1..100] do
            for j in [1..300] do
                C[100*(j-1)+i]:=[A[j]^2+A[j]*B[i]+B[i]^2,A[j],B[i]];
            od;
        od;

        for i in [1..30000] do
            if IsPrimeInt(C[i][1]) then
                if 2*C[i][2]>=-C[i][3] then
                    Add(D,C[i]);
                fi;
            fi;
        od;

        T:=Set(D);
        b:=Number(T);
        x:=Indeterminate(Rationals,"x");

        for i in [1..b]do
            if T[i][1]=a then
                return x^3-3*a*x-(2*T[i][2]+T[i][3])*a;
                break;
            fi;
        od;
    else
        return 1=2;
    fi;
end;

##########################################################
PolynomialOfM:=function(arg)
    local A,B,a1,a2,a3,b,c,d,e,i,j,k,x,u,w,y;
    B:=[];

    d:=arg[1];
    a1:=1;

    if Length(arg)=2 then
        a1:=arg[2];
    fi;

    a2:=a1;
    a3:=a1;

    if Length(arg)=4 then
```

```
        a1:=arg[2];
        a2:=arg[3];
        a3:=arg[4];
    fi;

    for b in [1..a1] do
        if B<>[] then
            break;
        fi;
        for c in [1..a2] do
            if B<>[] then
                break;
            fi;
            for e in [1..a3] do
                if B<>[] then
                    break;
                fi;
                y:=List([1..100],i->100*(b-1)+i);
                u:=List([1..100],i->100*(c-1)+i);
                w:=List([1..100],i->100*(e-1)+i);
                A:=[];

                for i in [1..100] do
                    if B<>[] then
                        break;
                    fi;
                    for j in [1..100] do
                        if B<>[] then
                            break;
                        fi;
                        for k in [1..100] do
                            if B<>[] then
                                break;
                            fi;
                            A[10000*(i-1)+100*(j-1)+k]:=[y[i],u[j],w[k]];
                        od;
                    od;
                od;

                x:=Indeterminate(Rationals,"x");
                for i in [1..1000000] do
                    if B<>[] then
                        break;
                    fi;
                    if Gcd(A[i][2],A[i][3])=1 then
                        if d*A[i][1]^2=4*A[i][2]*A[i][3]^3-27*A[i][2]^2 then
                            if A[i][3] mod 3 <> 0
                               or (A[i][3] mod 3 =0 and A[i][2]*A[i][3] mod 9 <>3 and
                                   (A[i][2] mod 9 =A[i][3]-1) or A[i][2] mod 9 = A[i][3]+1)
                               or (A[i][3] mod 3 =0 and A[i][2]*A[i][3] mod 9 =3 and
                                   (A[i][2] mod 27 =A[i][3]-1) or A[i][2] mod 27 = A[i][3]+1) then
                                if IsIrreducible(x^3-A[i][2]*A[i][3]*x-A[i][2]^2) then
                                    Add(B,A[i]);
                                fi;
                            fi;
                        fi;
                    fi;
                od;
            od;
        od;
    od;
    if B<>[] then
        return x^3-B[1][2]*B[1][3]*x-B[1][2]^2;
    else
        return B;
    fi;
end;

######################################################
MinimalPolynomial6:=function(l1,l2)
    local A1,A2,A3,B1,B2,B3,c0,c1,c2,c3,c4,c5;

    if IsPolynomial(l1) then
        l1:=CoefficientsOfUnivariatePolynomial(l1);
    fi;
    if IsPolynomial(l2) then
        l2:=CoefficientsOfUnivariatePolynomial(l2);
    fi;

    A1:=l1[3];
    A2:=l1[2];
    A3:=l1[1];
    B1:=l2[3];
    B2:=l2[2];
    B3:=l2[1];

    c5:=2*A1*B1;

    c4:=A1^2*B1^2+2*A1^2*B2+2*A2*B1^2-6*A2*B2;

    c3:=27*A3*B3-9*A1*A2*B3+2*A1^3*B3-9*A3*B1*B2-5*A1*A2*B1*B2+2*A1^3*B1*B2
        +2*A3*B1^3+2*A1*A2*B1^3;

    c2:=27*A1*A3*B1*B3-9*A1^2*A2*B1*B3+2*A1^4*B1*B3+9*A2^2*B2^2
        -6*A1^2*A2*B2^2+A1^4*B2^2-9*A1*A3*B1^2*B2-6*A2^2*B1^2*B2
        +3*A1^2*A2*B1^2*B2+2*A1*A3*B1^4+A2^2*B1^4;

    c1:=-81*A2*A3*B2*B3+27*A1^2*A3*B2*B3+27*A1*A2^2*B2*B3
        -15*A1^3*A2*B2*B3+2*A1^5*B2*B3+27*A2*A3*B1^2*B3-9*A1*A2^2*B1^2*B3
        +2*A1^3*A2*B1^2*B3+27*A2*A3*B1*B2^2-9*A1^2*A3*B1*B2^2
        -3*A1*A2^2*B1*B2^2+A1^3*A2*B1*B2^2-15*A2*A3*B1^3*B2
        +2*A1^2*A3*B1^3*B2+A1*A2^2*B1^3*B2+2*A2*A3*B1^5;

    c0:=-27*A2^3*B3^2+27*A1^2*A2^2*B3^2-9*A1^4*A2*B3^2+A1^6*B3^2
        -27*A1*A2*A3*B1*B2*B3+9*A1^3*A3*B1*B2*B3+18*A2^3*B1*B2*B3
```

```
        -9*A1^2*A2^2*B1*B2*B3+A1^4*A2*B1*B2*B3+9*A1*A2*A3*B1^3*B3
        -2*A1^3*A3*B1^3*B3-4*A2^3*B1^3*B3+A1^2*A2^2*B1^3*B3
        -27*A3^2*B2^3+18*A1*A2*A3*B2^3-4*A1^3*A3*B2^3-4*A2^3*B2^3
        +A1^2*A2^2*B2^3+27*A3^2*B1^2*B2^2-9*A1*A2*A3*B1^2*B2^2
        +A1^3*A3*B1^2*B2^2+A2^3*B1^2*B2^2-9*A3^2*B1^4*B2
        +A1*A2*A3*B1^4*B2+A3^2*B1^6;

return x^6-c5*x^5+c4*x^4-c3*x^3+c2*x^2-c1*x+c0;
end;

##########################################################
CompositionOfDefiningPolynomialsPlus:=function(a,b,c)
    local d1,d2,M1,M2,M3,i,j,k;

    if IsPolynomial(a) then
        a:=CoefficientsOfUnivariatePolynomial(a);
    fi;

    if IsPolynomial(b) then
        b:=CoefficientsOfUnivariatePolynomial(b);
    fi;

    d1:=Length(a)-1;
    d2:=Length(b)-1;

    M1:=NullMat(d1*d2,d1*d2);
    for i in [1,2..d1-1] do
        for j in [1,2..d1] do
            if i+1=j then
                M1[i][j]:=1;
            fi;
        od;
    od;

    for i in [1,2..d1] do
        M1[d1][i]:=-a[i];
    od;

    for i in [1,2..d2-1] do
        for j in [1,2..d1] do
            for k in [1,2..d1] do
                M1[i*d1+k][i*d1+j]:=M1[k][j];
            od;
        od;
    od;

    M2:=NullMat(d1*d2,d1*d2);

    for i in [1,2..(d2-1)*d1] do
        M2[i][d1+i]:=c;
    od;

    for i in [1,2..d1] do
        for j in [1,2..d2] do
            M2[d1*(d2-1)+i][i+d1*(j-1)]:=-b[j]*c;
        od;
    od;

    M3:=M1+M2;
    return CharacteristicPolynomial(M3);
end;

###########################################################
CompositionOfDefiningPolynomials:=function(a,b,c)
    local d1,d2,M1,M2,M3,i,j,k;

    if IsPolynomial(a) then
        a:=CoefficientsOfUnivariatePolynomial(a);
    fi;

    if IsPolynomial(b) then
        b:=CoefficientsOfUnivariatePolynomial(b);
    fi;

    d1:=Length(a)-1;
    d2:=Length(b)-1;

    M1:=NullMat(d1*d2,d1*d2);
    for i in [1,2..d1-1] do
        for j in [1,2..d1] do
            if i+1=j then
                M1[i][j]:=1;
            fi;
        od;
    od;

    for i in [1,2..d1] do
        M1[d1][i]:=-a[i];
    od;

    for i in [1,2..d2-1] do
        for j in [1,2..d1] do
            for k in [1,2..d1] do
                M1[i*d1+k][i*d1+j]:=M1[k][j];
            od;
        od;
    od;

    M2:=NullMat(d1*d2,d1*d2);

    for i in [1,2..(d2-1)*d1] do
        M2[i][d1+i]:=c;
    od;

    for i in [1,2..d1] do
```

```
        for j in [1,2..d2] do
            M2[d1*(d2-1)+i][i+d1*(j-1)]:=-b[j]*c;
        od;
    od;

    M3:=M1*M2;
    return CharacteristicPolynomial(M3);
end;

###########################################################
DiscriminantOfField:=function(a)
    local i,j,l,f,F,M,Mat,A;

    l:=Length(a);
    f:=UnivariatePolynomial(Rationals,a,1);
    F:=FieldByPolynomial(f);
    M:=MaximalOrderBasis(F);
    Mat:=[];

    for i in [1..l-1] do
        Mat[i]:=ExtRepOfObj(M[i]);
    od;

    A:=Discriminant(f)*(Determinant(Mat)^2);

    return A;
end;

###########################################################
DefinitionPolynomial4:=function(arg)
    local p,k0,k1,k2,f,a,k,n1,n0,m1,m0,i,j,i1,i0,o,A;

    p:=arg[1];
    k:=(p-1)/4;
    n1:=(k^3)*4;
    n0:=k^4;
    m1:=2*n1;
    m0:=2*n0;

    A:=[];

    i1:=Int(n1/p);
    i0:=Int(n0/p);

    if Length(arg)=3 then
        i1:=arg[2]; i0:=arg[3];
    fi;

    k0:=k^4 mod p;
    k1:=-4*k^3 mod p;
    k2:=(-p+k+1)/2;

    for i in [1..2*i1] do
        for j in [1..2*i0] do
            a:=[k0+p*(j-i0),k1+p*(i-i1),k2,1,1];
            f:=UnivariatePolynomial(Rationals,a,1);
            if IsIrreducible(f) then
                if DiscriminantOfField(a) = p^3 then
                    Add(A,f);
                    # Print(f,"\n");
                fi;
            fi;
        od;
    od;

    return A;
end;

###########################################################
DefinitionPolynomial5:=function(arg)
    local p,k0,k1,k2,k3,f,a,k,n2,n1,n0,m2,m1,m0,i,j,l,i2,i1,i0,o,A;

    p:=arg[1];
    k:=(p-1)/5;
    n2:=(k^3)*10;
    n1:=(k^4)*5;
    n0:=k^5;
    m2:=2*n2;
    m1:=2*n1;
    m0:=2*n0;

    A:=[];

    i2:=Int(n2/p);
    i1:=Int(n1/p);
    i0:=Int(n0/p);

    if Length(arg)=4 then
        i2:=arg[2];
        i1:=arg[3];
        i0:=arg[4];
    fi;

    k0:=-k^5 mod p;
    k1:=5*k^4 mod p;
    k2:=-10*k^3 mod p;
    k3:=-2*k;

    for l in [1..2*i2] do
        for i in [1..2*i1] do
            for j in [1..2*i0] do
                a:=[k0+p*(j-i0),k1+p*(i-i1),k2+p*(l-i2),k3,1,1];
                f:=UnivariatePolynomial(Rationals,a,1);
                if IsIrreducible(f) then
                    if DiscriminantOfField(a) = p^4 then
```

```
                        Add(A,f);
                        # Print(f,"\n");
                    fi;
                fi;
            od;
        od;
    od;

    return A;
end;

############################################################
MatB:=function(P,p)
    local k,B,l,i1,i2,i3,i4,i5,i6,i7,i8,i9,i10,i11,i12,f1,f2,fL,F1,F2;

    B:=[];
    l:=Length(P);
    k:=(p-1)/l;

    for i1 in [1..l+1] do
        B[i1]:=[];
    od;

    for i2 in [1..l+1] do
        for i3 in [1..l+1] do
            B[i2][i3]:=[];
        od;
    od;

    for i4 in [1..l] do
        for i5 in [1..l] do
            for i6 in [1..l] do
                f1:=CompositionOfDefiningPolynomials(P[i4],P[i5],1);
                F1:=Factors(f1);
                f2:=CompositionOfDefiningPolynomials(F1[1],P[i6],1);
                F2:=Factors(f2);
                fL:=CoefficientsOfUnivariatePolynomial(F2[1]);
                B[i4+1][i5+1][i6+1]:=(k^2-fL[1])/p;
            od;
        od;
    od;

    for i7 in [1..l+1] do
        for i8 in [1..l+1] do
            if i7=i8 then
                B[1][i7][i8]:=1;
            else
                B[1][i7][i8]:=0;
            fi;
        od;
    od;

    for i9 in [1..l] do
        for i10 in [1..l+1] do
            if i9+1=i10 then
                B[i9+1][1][i10]:=1;
            else
                B[i9+1][1][i10]:=0;
            fi;
        od;
    od;

    for i11 in [1..l] do
        for i12 in [1..l+1] do
            if i11+1=i12 then
                B[i11+1][i12][1]:=k;
            else
                B[i11+1][i12][1]:=0;
            fi;
        od;
    od;

    return B;
end;

############################################################
```

# References


[1] E. Bannai and T. Ito, *Algebraic combinatorics. I*, The Benjamin/Cummings Publishing Co. Inc., Menlo Park, CA, 1984.

[2] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.8*, 2016, (`http://www.gap-system.org`).

[3] A. Hanaki, *Association schemes of prime order and their splitting fields*, Sendai Mini Symposium on Number Theory and Combinatorial Theory 2005 (Tohoku University), Jan 30–31, 2006, (in Japanese), `http://www.math.is.tohoku.ac.jp/~taya/sendaiNC/2005/report/hanaki.pdf`.

[4] T. Komatsu, *Tamely ramified eisenstein fields with prime power discriminants*, Kyushu J. Math. **62** (2008), 1–13.

[5] ———, *On algebraic number fields unramified outside a prime number and Galois closure fields*, Japan-Korea Workshop on Algebra and Combinatorics (Kyushu University), Oct 21–22, 2006.

[6] S. Teranishi, *On non-Galois unramified extensions of number fields*, Master's thesis, Shinshu University, 2009, (in Japanese).