# Continuum limit for Laplace and elliptic operators on lattices

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This talk is devoted to continuum limit problems of discrete Schrödinger operators on lattices, which is based on a joint work with Keita Mikami (RIKEN) and Shu Nakamura (Gakushuin University).

The first part of this talk concerns the asymptotic behaviors of Laplace operators $H_h = H_{0,h} + V$ on general lattices $h\Lambda \subset \mathbb{R}^d$ perturbed with a real-valued potential $V$ on $\mathbb{R}^d$ as the mesh size $h$ tends to zero. We show that, when $\Lambda$ is equipped with a graph structure, $H_h$ converges to $H = H_0 + V$ in the generalized norm resolvent sense as $h \to +0$ under the same assumption on $V$ as in our previous work Nakamura-Tadano (2021, JST), where $H_0 = p_0(D_x)$ is the second order elliptic operator associated to the given graph structure. It is also shown that, as an application of the above result, the continuum limit of the Laplacian on the hexagonal lattice $\Lambda_{\mathrm{hex}}$, which is not a lattice in that $\Lambda_{\mathrm{hex}} \ncong \mathbb{Z}^2$, is $-\frac{3}{4}\Delta$ in the above-mentioned sense.

In the second part of this talk, we study the discretization $H_h$ of a second order strictly elliptic operator $H$ on $\mathbb{R}^d$ onto the square lattice $h\mathbb{Z}^d$. We show that $H_h$ converges to $H$ as $h \to +0$ in the generalized norm resolvent topology.