ABSTRACTS

2024 RIMS symposium: Mathematical aspects of quantum fields and related topics

---

👤 Ryokichi Tanaka

📝 Noise sensitivity and stability on groups (ランダムウォークのノイズ鋭敏性と安定性)

Abstract

Noise sensitivity problem asks the following:
Given a stochastic process defined in terms of i.i.d. sequences, does a small number of resamplings produce an independent copy of the original one asymptotically?
In this talk, we ask the following questions:
Is a random walk on a group noise sensitive? What group admits a noise sensitive random walk? Does mathematical study on this problem say anything about our real life?

（ノイズ鋭敏性の問題とは独立同分布な確率変数列を用いて定義される確率過程に対して、数少ない再サンプリングにより漸近的に元の過程と独立な過程が得られるかを問うものです。さて有限生成群上のランダムウォークはノイズ鋭敏的なのでしょうか。あるいはどのような群を考えるとノイズ鋭敏性を持つランダムウォークが得られるのでしょうか。他の確率モデルでも同様な問題が考えられるでしょうか。この問題の数学的な考察は現実世界の現象に何らかの洞察をもたらすのでしょうか。）

👤 Shousuke Ohmori

📝 On a rigged Hilbert space approach for quasi-Hermitian composite systems Abstract

Abstract

→ See the last page.

👤 Oliver Matte

📝 Low energy spectrum of a polaron in a weak constant magnetic field

Abstract

We consider the Fröhlich Hamiltonian for a polaron in a constant magnetic field, which is translation invariant in the z-direction. For weak magnetic field strengths, we show that the low-lying spectra of the corresponding fiber Hamiltonians for fixed, sufficiently small total momenta in the z-direction approximately have Landau level structures. The spacing of the Landau levels is determined by the renormalized polaron mass. A key technique in the proof, borrowed from the theory of periodic Schrödinger operators coupled to weak magnetic fields, is the construction of an effective Hamiltonian acting in the sub-Hilbert space generated by a system of magnetic quasi-Wannier functions for the polaron. The talk is based on joint work with Horia Cornean und Rohan Ghanta.

👤 Hajime Moriya

📝 On the Thermal Area Law

Abstract

The area law has played an important role in both condensed matter physics and quantum field theory. In this work, we propose two possible versions of the thermal area law within an infinite-dimensional framework.

👤 Nobuhiro Asai

📝 Combinatorial Moment Formula with Two Parameters Derived from Non-Commutative Probabilistic Viewpoint

Abstract

In this talk, we shall introduce two parameterized deformation of the classical Poisson random variable from the viewpoint of noncommutative probability, namely (q, s)-Poisson type operator on the two parameterized deformed Fock space, namely, the (q, s)-Fock space constructed by the weighted q-deformation approach. The recurrence formula for the orthogonal polynomials of the (q, s)-deformed Poisson distribution is determined. Moreover we shall also give the combinatorial moment formula of the (q, s)-Poisson type operator by using the set partitions and the card arrangement technique with their

statistics. Our method presented in this talk provides nice combinatorial interpretations to parameters, q and s.

👤 Zied Ammari

📝 Bose-Hubbard model and classical Kubo-Martin-Schwinger condition

Abstract

The classical Kubo-Martin-Schwinger (KMS) condition is a fundamental property of statistical mechanics which characterize thermal equilibria of infinite classical mechanical systems. It was introduced in the seventies by Gallavotti and Verboven. In this talk I will explore this concept in the framework of Bose-Hubbard model and discrete non-linear Schrödinger equation over finite graphs and study the high temperature limit.

👤 Benjamin Hinrichs

📝 The Ising Phase Transition in the Spin Boson Model

Abstract

Feynman-Kac formulas are well-known to provide a connection between the spin boson model, describing a spin linearly coupled to a bosonic quantum field, and a continuous one-dimensional Ising model. If the spin boson model is infrared-divergent, i.e., if the bosons are massless, then the pair interaction of the Ising model decays slowly. We discuss how long range order of the Ising correlation functions then implies absence of ground states for the spin boson model. Results implying long range order at large spin-field coupling are also presented. This talk is based on joint work with Volker Betz, Mino Kraft and Steffen Polzer.

👤 Yusuke Arike

📝 Modular linear differential equations and vertex algebras

Abstract

Modular linear differential equations (MLDEs) are differential equations involving Serre derivatives, with coefficients that are modular forms. In this talk, I will explain the role of

MLDEs in the theory of vertex algebras and discuss how they can be used in the classification of vertex algebras. Specifically, I will present our result on the classification of vertex algebras whose characters are solutions of third-order MLDEs.
This talk is based on joint work with Kiyokazu Nagatomo and Yuichi Sakai.

👤 Fumio Hiroshima

📝 TBA

Abstract:TBA

👤 Alberto Ibort

📝 On the categorical foundations of field theories: classical and quantum

Abstract

We will argue that the categorical/groupoidal setting for quantum mechanical systems provides a natural background for classical and quantum field theories. From this perspective fields are just functors between appropriate categories (groupoids). The space of fields of the theory become a category of (pre)sheaves and they provide natural instances of topos. Some natural explicit examples, including topological gauge theories and other relevant models, will be discussed.

👤 Hayato Saigo

📝 Towards Mathematical Principles of Quantum Fields: Category Algebras and States on Categories

Abstract

Historically, quantum field theory emerged as a unification of relativity and quantum mechanics. The former can be seen as a categorical structure, and the latter as a noncommutative probability structure. In this talk, we will introduce an attempt at quantum field theory based on the concept of "category algebras," which are convolution algebras defined on categories as spacetime, and "states on categories," which are positive unital linear functionals on them. (Based in part on joint research with Hiroshi Ando, Soichiro Fujii, Takahiro Hasebe, Kazuya Okamura, and Izumi Ojima.)

👤 Kazuya Okamura

📝 Quantum physics from the perspective of categories of quantum instruments

Abstract

The speaker gives a talk on quantum physics using quantum instruments. We define categories of quantum instruments and clarify their properties. We then discuss how utilizing them enables us to describe a wide class of quantum systems. Here, we suppose quantum fields and thermodynamic systems as concrete examples.

👤 Janik Kruse

📝 Asymptotic Completeness in Quantum Scattering Theory - Mourre Theory and Asymptotic Observables in Relativistic QFT

Abstract

The goal of scattering theory is to understand how a quantum system of interacting particles evolves asymptotically. A central concept in scattering theory is asymptotic completeness, which asserts that every state can be decomposed into a bound and a scattering state. While asymptotic completeness is well understood in non-relativistic quantum mechanics, it remains an open and challenging problem in local relativistic quantum field theory due to various conceptual and technical complications. In quantum mechanics, many proofs of asymptotic completeness depend on the convergence of asymptotic observables. In QFT, Huzihiro Araki and Rudolf Haag (1967, doi:10.1007/BF01645754) identified particle detectors as natural asymptotic observables. They demonstrated the convergence of these detectors on scattering states, but the convergence on arbitrary states, which is relevant for establishing asymptotic completeness, remains unproven. Relatively recently, Wojciech Dybalski and Christian Gérard (2014, doi:10.1007/s00220-014-2069-y) made progress in this area by adapting quantum mechanical propagation estimates to QFT. They covered coincident arrangements of multiple detectors sensitive to particles with distinct velocities, but they did not manage to establish the convergence of a single detector due to a missing low-velocity estimate. Typically, such an estimate is proved through Mourre's conjugate operator method, a powerful tool in quantum mechanics that has so far resisted adaptation to QFT. In a recent paper (2024, doi:10.1007/s00220-024-05091-7), we succeeded in applying Mourre's method to QFT through Haag-Ruelle scattering theory. This allowed us to prove the convergence of a single Araki-Haag detector on states of bounded energy below the three-particle threshold.

👤 Tomoyuki Shirai

📝 Accumulated spectrograms for hyperuniform determinantal point processes

Abstract

We introduce the concept of determinantal point processes (DPPs) associated with locally trace-class self-adjoint operators and also the notion of hyperuniformity of point processes. For a given bounded set and a locally trace class operators, one can define the accumulated spectrogram, which is closely related to the density of points in the DPP. The spectrogram is defined through the short-time Fourier transform which is often used in the time-frequency analysis. We then present a convergence theorem for accumulated spectrograms along an exhaustion formed by dilations of a bounded set, which can be viewed as a version of the well-known circular law observed in the Ginibre point process. This talk is based on a joint work with Pierre Lazag and Makoto Katori.

👤 Marco Falconi

📝 Binding by strong particle-field interactions

Abstract

In this talk I review recent results concerning enhanced binding for particle-field systems: a bound state is induced, by a strong particle-field interaction, on a composite system in which the particle subsystem alone does not have bound states.
In particular, I will focus on a result together with A. Olgiati and N. Rougerie, in which enhanced binding is obtained in the strong coupling limit by a combination of concentration-compactness and semiclassical techniques.

👤 Masanao Ozawa

📝 Order Relations of Quantum Observables Derived by Quantum Set Theories with Different Quantum Conditionals

Abstract

The projection lattice of the Hilbert space describing a quantum system is called "quantum logic" and defines a logic that does not satisfy the distributive law. Based on the quantum logic, a set theory can be developed, which is called "quantum set theory". Quantum set

theory allows us to develop mathematics based on quantum logic. More precisely, to each proposition in classical mathematics, it is possible to assign the projection-valued truth value in quantum logic. This is called "quantum mathematics". The object x such that the truth value 1 is assigned to the classical mathematical proposition "x is a real number", i.e., a real number defined in quantum mathematics, is called a "quantum real number". In 1979, Gaisi Takeuti suggested the fact that there is a one-to-one correspondence between quantum real numbers and quantum observables (of the quantum system described by the quantum logic under consideration), which has been proven under the current formulation of "quantum set theory". Then, the projection-valued truth value of the relation between real numbers defined in quantum mathematics can be transferred between corresponding quantum observables. For example, for quantum observables X and Y, the projection-valued truth value of the equality relation X=Y is the greatest lower bound of the projections assigned to the proposition "X≤r ⟺Y≤ r" over all rational numbers r. It is known that this coincides with the projection onto the subspace {φ∈H| f(X)φ=f(Y)φ for all f} of the Hilbert space H. Similarly, the projection-valued truth value ⟦X≤Y⟧ of the order relation X≤Y for quantum observables X and Y is given as the lower bound of the projections assigned to the proposition "Y≤r⇒X≤ r" over all rational numbers r. However, here is a problem about quantum conditionals. In classical logic, the conditional "P⇒Q" (if P then Q) is defined as "(not P) or Q", but in quantum logic, it is known that this definition is insufficient, and currently, three definitions for conditional are proposed as reasonable. In this presentation, I plan to report on the following:
(1) For these three conditionals, the truth value assignment for the equality relation is unique, whereas for the order relation, different projections are assigned as the truth value of "X≤Y".
(2) The relation "X≾Y" between X and Y defined as ⟦X≤Y⟧=1 is equivalent to the "spectral order" between X and Y introduced by Olson,
(3) The meaning of the truth value ⟦X≤Y⟧ can be tested by a physical experiment; comparing the order relation of the measured values x and y obtained from successive projective measurements of X and Y performed in a state φ where "X≤Y" holds, that is, in a state φ where ⟦X≤Y⟧φ=φ holds.

👤 Hiroshi Ando

📝 Large scale geometry of Banach-Lie groups

Abstract

In this talk, we report our work on the large scale geometric study of Banach-Lie groups, especially of linear Banach-Lie groups. We show that the exponential length, originally introduced by Ringrose for unitary groups of C^*-algebras, defines the quasi-isometry type of any connected Banach-Lie group. We also give an affirmative answer to Rosendal's question regarding the existence of a minimal metric for connected Banach-Lie groups. This is a joint work with Michal Doucha (Czech Academy of Sciences), and it is an extension of our previous work with Doucha and Yasumichi Matsuzawa (Shinshu University).

On a rigged Hilbert space approach for quasi-Hermitian composite systems

Shousuke Ohmori

National Institute of Technology, Gunma College
Waseda Research Institute for Science and Engineering, Waseda University

Abstract

A rigged Hilbert space (RHS), also known as Gel'fand's triplet, was first introduced by I.M.Gel'fand and his collaborators to bridge distribution theory and Hilbert space theory. This space has been also studied to provide a rigorous mathematical formulation for Dirac's bra-ket notation. In fact, the nuclear spectral theorem of RHS shows the spectral expansions of the bra and ket vectors through the (generalized) eigenvectors of the observables, and the Dirac's bra-ket formalism can be formulated from the spectral expansions. In recent years, RHS approach has been found to be effective in the mathematical treatment of modern quantum physics, such as non-equilibrium open systems and non-Hermite quantum systems. For instance, in the problem of a quantum damped system, the Hamiltonian exhibits only real spectra in the L^2-space. However when the RHS is selected as the fundamental space, the Hamiltonian exhibits complex eigenvalues, which are interpreted as the resonant states. As in this case, we often encounter the physical problems that can not be addressed within the framework of the Hilbert space alone.

In my presentation, I focus on a non-Hermitian composite system whose non-Hermite operator possesses a characteristic symmetric relation, $A^\dagger = \eta A \eta^{-1}$, where η is a positive operator, called the intertwining operator. This type of operator is called a quasi-Hermitian operator. We propose an RHS suitable for this system where the obtained RHS is utilized to construct the bra and ket vectors and produce the spectral decomposition for the quasi-Hermitian operator. We also show that the symmetric relations regarding quasi-Hermitian operators can be extended to dual spaces, and all descriptions based on the bra-ket formalism are completely developed in the dual spaces. Using these dual spaces, the issue of defining the adjoint of a quasi-Hermitian operator in non-Hermitian composite systems is tackled. Finally, we talk the application of our methodology to a non-Hermitian harmonic oscillator composed of conformal multi-dimensional many-body systems.