On a rigged Hilbert space approach for quasi-Hermitian composite systems

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Abstract

A rigged Hilbert space (RHS), also known as Gel'fand's triplet, was first introduced by I.M.Gel'fand and his collaborators to bridge distribution theory and Hilbert space theory. This space has been also studied to provide a rigorous mathematical formulation for Dirac's bra-ket notation. In fact, the nuclear spectral theorem of RHS shows the spectral expansions of the bra and ket vectors through the (generalized) eigenvectors of the observables, and the Dirac's bra-ket formalism can be formulated from the spectral expansions. In recent years, RHS approach has been found to be effective in the mathematical treatment of modern quantum physics, such as non-equilibrium open systems and non-Hermite quantum systems. For instance, in the problem of a quantum damped system, the Hamiltonian exhibits only real spectra in the L^2-space. However when the RHS is selected as the fundamental space, the Hamiltonian exhibits complex eigenvalues, which are interpreted as the resonant states. As in this case, we often encounter the physical problems that can not be addressed within the framework of the Hilbert space alone.

In my presentation, I focus on a non-Hermitian composite system whose non-Hermite operator possesses a characteristic symmetric relation, $A^\dagger = \eta A \eta^{-1}$, where η is a positive operator, called the intertwining operator. This type of operator is called a quasi-Hermitian operator. We propose an RHS suitable for this system where the obtained RHS is utilized to construct the bra and ket vectors and produce the spectral decomposition for the quasi-Hermitian operator. We also show that the symmetric relations regarding quasi-Hermitian operators can be extended to dual spaces, and all descriptions based on the bra-ket formalism are completely developed in the dual spaces. Using these dual spaces, the issue of defining the adjoint of a quasi-Hermitian operator in non-Hermitian composite systems is tackled. Finally, we talk the application of our methodology to a non-Hermitian harmonic oscillator composed of conformal multi-dimensional many-body systems.