Mathematical Aspects of Quantum Fields and Related Topics

25-27 June 2025

Abstracts

Dates: 25-27 June 2025
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Organizers: Itaru Sasaki (Shinshu U.), Hayato Saigo (ZEN U.)

Program

Wednesday, 25 June

Shuji Watanabe (Sanjo City U.)
The BCS model of superconductivity with external magnetic field

We deal with a type I superconductor in a constant external magnetic field. We construct the BCS model of superconductivity with external magnetic field and study the BCS gap equation obtained. We show that there is a unique magnetic field (the critical magnetic field) given by a smooth function of the temperature and that there is also a unique solution (the gap function) given by a smooth function of both the temperature and the external magnetic field. Using the grand potential, we show that the transition from the normal state to the superconducting state in a type I superconductor is of the first order. Moreover, we obtain the explicit expression for the entropy gap.

Takuya Mine (Kyoto Institute of Technology)
3D point interactions on α-determinantal point processes

In this talk, we consider the 3-dimensional Schrödinger operators with point interactions supported on the α-determinantal point processes, and explain recent progress about the analysis of this operator, e.g., the self-adjointness, the spectrum, and the asymptotics of the integrated density of states (IDS) $N(\lambda)$ as $\lambda \to -\infty$. The case $\alpha = 0$ (Poisson point process) is analyzed by the speaker in a joint work with M. Kaminaga and F. Nakano, and the case $\alpha = -1$ (determinantal point process) is also studied by the speaker. We shall show how the obtained results are generalized for more general α. This talk is based on a joint work with Professor Tomoyuki Shirai (Kyushu University).

Yoshinori Kamijima (Toyo U.)
Lace expansion for the quantum Ising model

The classical Ising model is one of the models of ferromagnetism. It is well-known that this exhibits critical phenomena. For example, the susceptibility χ diverges at a critical point β_{c} and behaves asymptotically as a power-law $(\beta_{\mathrm{c}} - \beta)^{-\gamma}$ in the vicinity of the critical point. This exponent γ, referred to a critical exponent, is believed to be universal such as the fact that it takes the value 1 in lattice dimensions $d \geq 4$. The quantum Ising model is obtained by introducing a transverse field q, which controls the quantum effect, to the classical one, whereby the spin variables become operators. Phase transitions are caused not only by thermal fluctuations but also by quantum fluctuations. We are interested in the robustness of critical phenomena under such a quantum perturbation.

In this talk, I will explain derivation of a lace expansion for the quantum Ising model. It is well-known that a lace expansion for the classical Ising model was derived in [Sakai (2007) *Commun. Math. Phys.*]. That work utilized the random-current representation [Aizenman (1982) *Commun. Math. Phys.*]. Similarly, our approach is based on the random-current representation [Björnberg and Grimmett (2009) *J. Stat. Phys.*] [Crawford and Ioffe (2010) *Commun. Math. Phys.*] for the quantum Ising model, which allows expressions for expectation values of operators to be translated into the language of Poisson point processes. The expansion coincide with a sequel to another lace expansion for the classical Ising model [Kamijima-Sakai (2025) arXiv:2501.06592] as the transverse-field q goes to 0.

This talk is based on joint work with Akira Sakai (Hokkaido University, Japan).

Yuki Tsujimoto (Kyushu U.)
The evaluation of the spatial decaying of the ground state of the Pauli-Fierz model from below

In this talk, we evaluate the spatial decay of the ground state Ψ_0 of the Pauli-Fierz Hamiltonian from below by using a path integral method. The evaluation from above is known [1], but it from below is not known and non-trivial. It can be obtained by using photon number rotations Θ and a geodesic distance. As a result, the order of the spatial decay of the ground state Ψ_0 is determined.

Reference: [1] Takeru, Hidaka and Fumio Hiroshima. "Pauli-Fierz model with Kato-class potentials and exponential decays". Reviews in Mathematical Physics, 22, 1181–1208, 2010.

Masanao Ozawa (Ritsumeikan U.)
Time in Macroscopic Extension of Quantum Theory: Harmonic Oscillators

The Hermitian time operator does not exist on the Hilbert space of quantum states, but the optimum POVM may exist that describes the optimal measurement of time. By the Naimark theorem, this POVM can be extended to a projection-valued measure representing a Hermitian operator on a larger Hilbert space. We consider the physical meaning of the extended Hilbert space and the dilated Hermitian operator. In this talk, we study the phase of a harmonic oscillator as time in quantum mechanics to suggest that the extended Hilbert space represents classical mechanics and that the optimum time POVM extends to a classical time observable.

Von Neumann's uniqueness theorem concludes the absence of a Hermitian phase operator, while quantum estimation theory provides the optimal POVM for phase measurement. We construct a Naimark dilation of the optimal POVM by adding quantum states with infinite excitations to the original Hilbert space by a mathematically rigorous way based on non-standard analysis, so that we obtain a Hermitian operator on the extended Hilbert space. We show that this Hilbert space represents classical mechanics and the dilation of the phase POVM coincide with the classical time operator. Thus, the optimal time POVM is the projection of classical time operator onto the quantum subspace. We discuss the quantization as the projection from the classical Hilbert space onto the quantum subspace and consider why time is not a quantum observable.

Izumi Ojima (RODreP)
TBA

Thursday, 26 June

Tadahiro Miyao (Hokkaido U.)
Hall Conductivity in Infinitely Extended Gapped Fermionic Systems

We study an infinitely extended system of fermions on a d-dimensional lattice, subject to short-range, (magnetic) translation-invariant interactions. Assuming that the system possesses a gapped ground state, we investigate its current response to a constant external electric field. Using the framework of non-equilibrium almost-stationary states (NEASS), we prove that the longitudinal current density induced by a static electric field of strength ε is of order $O(\varepsilon^\infty)$, indicating that the system behaves as an insulator in the conventional sense. In contrast, we show that the Hall current density is linear in ε up to corrections of order $O(\varepsilon^\infty)$. The proportionality constant, defined as the Hall conductivity σ_H, is given by a generalization of the well-known double commutator formula, extended here to interacting systems. As a consequence of our analysis, we find that σ_H remains constant within a given gapped phase. Furthermore, in the two-dimensional case ($d = 2$), we prove that the Hall conductivity is quantized for invertible systems.

Benoit Collins (Kyoto U.)
Around the norm of generators the sum of epsilon-free elements and their matrix models

We consider a natural problem in tensor random matrix theory, that consists in trying to understand the sum of independent GUE with a tensor structure. We give new estimates for their norms and that of their limiting objects in terms of epsilon freeness. Part of this work is joint with Wangjun Yuan, and part is joint with Akihiro Miyagawa. After reviewing motivations, we will also mention recent related work by other authors in this direction.

Ryosuke Sato (Hokkaido U.)
CAR algebras and stochastic dynamics on point processes

In this talk, we will investigate stochastic processes on point configurations in a given discrete countable space, which is a mathematical model of random interacting particle systems. As is known, an operator algebra generated by fermionic creation and annihilation operators, called a CAR algebra, is intimately related to random point configurations. In this talk, we establish this connection at the dynamic level. Hence, we will discuss the KMS condition for states on a CAR algebra. In particular, we demonstrate that the analytic continuation of time evolution under KMS states gives rise to stochastic processes on point configurations. This idea provides a novel framework for constructing and analyzing such stochastic dynamics.

Yuma Takabayashi (Kyushu U.)
Functional integral approach to the ground state of the Pauli-Fierz model in non-relativistic quantum electrodynamics

We consider the Pauli-Fierz model, which describes a physical system of non-relativistic and spinless electrons minimally coupled to a quantized radiation field. The existence problem of the ground state of the Pauli-Fierz model is a central problem in the mathematical analysis of quantum field models. Bach, Fröhlich, and Sigal (1998) proved this problem using a renormalization group method for sufficiently small coupling constant without an infrared cutoff. Gérard (2000, 2006) gave a proof based on functional analytic point of view. Griesemer, Lieb, and Loss (2001) proved it for arbitrary coupling constant using the binding condition of the Pauli-Fierz model.

Kornikar Sen (Universidad Complutense de Madrid)
Energy extraction from quantum batteries and the role of entanglement in it

Miniaturised batteries, where quantum phenomena can be effective, are being extensively explored because of their portability and flexibility. We examine various methods of energy extraction from a quantum battery and try to draw a comparative picture, and find out how the performance of these methods depends on the quantum mechanical resources, such as entanglement. When a bipartite battery is in hand, one faces the dichotomy of choosing between global and local operations for energy extraction. On one hand, local operations are easy to implement, but on the other hand, local operations are usually less efficient than global ones. We compare the effectiveness of local and global operations in the context of energy extraction from quantum batteries. In this context, we define the locally passive states, the states from which local unitary operations are unable to perform energy extraction. We try to determine how the entanglement of the shared battery affects the locally and globally extractable energies from a quantum battery. Furthermore, we move from unitary to a more general method of energy extraction, that is, by using positive maps. We compare the amounts of extractable energy from a bipartite quantum battery by operating completely positive trace-preserving (CPTP) maps and non-completely positive trace-preserving

(NCPTP) maps, which are certainly positive and physically valid. The maps are always applied to a particular part of the shared battery. We find that these two different extractable energies show an analogy with the two different entanglement measures, viz., distillable entanglement and entanglement cost.

Isao Ishikawa (Kyoto U.)
Data-driven estimation of the Koopman operator in reproducing kernel rigged Hilbert spaces and its applications

In this talk, I will introduce a recently developed data-driven method for estimating the Koopman operator defined on a reproducing kernel Hilbert space (RKHS). By considering a suitable structure (called a Gelfand triple) built from the dynamical system, we can obtain a meaningful finite-dimensional approximation of the Koopman operator and a clear mathematical interpretation in a data-driven setting. I will also explain how this approach can be applied to detect equilibrium points and to reconstruct the original dynamical system from discrete data.

Friday, 27 June

Kenta Kojin (Japan Coast Guard Academy)
Introduction to non-commutative function theory

Noncommutative functions are generalizations of free polynomials. Domains of them are d-tuples of matrices. Therefore, their restriction to 1-by-1 matrices, hence scalars, are usual functions in d-variables. Since noncommutative functions have information on matrices of all sizes, they may have different properties from those of (commutative) functions. For example, if the noncommutative Jacobian is invertible for arbitrary points, then such a function must be globally injective rather than locally. This talk will begin with the definition of noncommutative functions and present some basic ideas and results.

Noriyoshi Sakuma (U. Osaka)
On the free gamma distributions

In this talk, we introduce a class of generalized Meixner-type free gamma distributions, which includes the free gamma distributions introduced by Anshelevich and certain scaled free beta prime distributions introduced by Yoshida. We present basic properties and mixture structures of these distributions.

Etsuo Segawa (Yokohama National U.)
Graph resonance induced by quantum walks

We consider a discrete-time quantum walk model on graphs which is tunable to the strength of the interaction with the outside by a parameter ϵ. In this talk, we characterize the stationary state for small ϵ using the spectrum information of the original graph. We show that an appropriate frequency, which is a complex value, provides the perfect transmitting even in the small ϵ.

Kazuya Okamura (Chubu U.)
On Bohr' s complementarity and quantum instruments

We present on the mutual contributions of Bohr's complementarity principle and quantum measurement theory from a mathematical perspective. The complementarity principle is an innovative approach to "scientific analysis and synthesis." Bohr proposed this principle in the late 1920s and refined it through his debates with Einstein and beyond. However, Bohr' s position has remained consistent since its original proposal. In this talk, we show that both the analytical and synthetic aspects proposed by the principle of complementarity can be given precise mathematical formulations. The necessary concepts for this formulation are provided by the framework of algebraic quantum theory and by quantum measurement theory based on quantum instruments.

Laura Elena Gonzalez Bravo (Universidad Carlos III de Madrid)
The category of non-commutative probabilities in Information Geometry

The foundational theorems of Cencov and Petz provide a characterization of invariant geometries for finite-dimensional classical and quantum systems, respectively. However, extending these insights into a unified framework that seamlessly incorporates infinite-dimensional and continuous systems remains a significant challenge. This work introduces the category of non-commutative probabilities, NCP, as a new foundation for information geometry. Built upon the language of operator algebras, NCP provides a common environment for both classical and quantum theories, applicable in both finite and infinite dimensions. Within this setting, we reformulate the problem of characterizing invariant geometries. Instead of classifying metric tensors on manifolds of states, we propose the classification of "fields of covariances," defined as functors from NCP to the category of Hilbert spaces. This approach recasts the crucial monotonicity property as a natural condition of functoriality. Furthermore, we demonstrate how this framework accommodates the description of statistical models as dedicated subcategories.

Hayato Saigo (ZEN U.)

Process, causality and noncommutative probability: A category-theoretic framework for quantum field theory

Starting from a category with events as objects and processes as arrows, we construct a category of "causality" from that category. Furthermore, to connect the structure of causality with quantitative and probabilistic structures, we consider noncommutative probability on categories through the concepts of category algebras and states on categories. We will discuss the relationship between this framework and foundations of quantum field theories.