

微分積分学 I 演習問題 3

問題 1. 以下の極限を求めよ.

- (1) $\lim_{x \rightarrow +\infty} \frac{x^2 + x + 1}{x^2 - x + 2}$
- (2) $\lim_{x \rightarrow +\infty} \frac{x^3 - 3x^2 + 2x + 5}{2x^3 + x^2 - x + 4}$
- (3) $\lim_{x \rightarrow +\infty} \frac{1 - e^x}{1 + e^x}$
- (4) $\lim_{x \rightarrow +\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$
- (5) $\lim_{x \rightarrow +\infty} \sqrt{x}(\sqrt{x+2} - \sqrt{x+1})$
- (6) $\lim_{x \rightarrow +\infty} \sqrt{x}(\sqrt{2x+3} - \sqrt{2x+1})$
- (7) $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{2}} - (1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{3}} - (1-x)^{\frac{1}{3}}}$
- (8) $\lim_{x \rightarrow 0} \frac{(1+3x)^{\frac{1}{5}} - (1-2x)^{\frac{1}{5}}}{(1+5x)^{\frac{1}{3}} - (1+2x)^{\frac{1}{3}}}$
- (9) $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + x + 1} - \sqrt{x^2 - x + 3}}{x - 1}$
- (10) $\lim_{x \rightarrow 1} \frac{\sqrt{4x^2 + 2x + 3} - \sqrt{4x^2 + x + 4}}{x - 1}$

解答:

$$(1) 1 \quad (2) \frac{1}{2} \quad (3) -1 \quad (4) 1 \quad (5) \frac{1}{2}$$

$$(6) \frac{1}{\sqrt{2}} \quad (7) \frac{3}{2} \quad (8) 1 \quad (9) \frac{1}{\sqrt{3}} \quad (10) \frac{1}{6}$$

$$(1) \frac{x^2 + x + 1}{x^2 - x + 2} = \frac{1 + \frac{1}{x} + \frac{1}{x^2}}{1 - \frac{1}{x} + \frac{2}{x^2}} \rightarrow 1.$$

$$(2) \frac{x^3 - 3x^2 + 2x + 5}{2x^3 + x^2 - x + 4} = \frac{1 - \frac{3}{x} + \frac{2}{x^2} + \frac{5}{x^3}}{2 + \frac{1}{x} - \frac{1}{x^2} + \frac{4}{x^3}} \rightarrow \frac{1}{2}.$$

$$(3) \frac{1 - e^x}{1 + e^x} = \frac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 1} \rightarrow -1.$$

$$(4) \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}} \rightarrow 1.$$

$$(5) \frac{\sqrt{x}(\sqrt{x+2} - \sqrt{x+1})}{\sqrt{x+2} + \sqrt{x+1}} = \frac{\sqrt{x}}{\sqrt{1 + \frac{2}{x}} + \sqrt{1 + \frac{1}{x}}} \rightarrow \frac{1}{2}.$$

$$(6) \frac{\sqrt{x}(\sqrt{2x+3} - \sqrt{2x+1})}{\sqrt{2 + \frac{3}{x}} + \sqrt{2 + \frac{1}{x}}} = \frac{2}{\sqrt{2}(\sqrt{2 + \frac{3}{x}} + \sqrt{2 + \frac{1}{x}})} \rightarrow \frac{1}{\sqrt{2}}.$$

$$(7) \frac{(1+x)^{\frac{1}{2}} - (1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{3}} - (1-x)^{\frac{1}{3}}} = \frac{2x}{2x} \cdot \frac{(1+x)^{\frac{2}{3}} + (1+x)^{\frac{1}{3}}(1-x)^{\frac{1}{3}} + (1-x)^{\frac{2}{3}}}{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}} \rightarrow \frac{3}{2}.$$

$$(8) \frac{(1+3x)^{\frac{1}{5}} - (1-2x)^{\frac{1}{5}}}{(1+5x)^{\frac{1}{3}} - (1+2x)^{\frac{1}{3}}} = \frac{5x}{3x} \cdot \frac{\sum_{i=1}^3 (1+5x)^{\frac{3-i}{3}} (1+2x)^{\frac{i-1}{3}}}{\sum_{i=1}^5 (1+3x)^{\frac{5-i}{5}} (1+2x)^{\frac{i-1}{5}}} \rightarrow 1.$$

$$(9) \frac{\sqrt{x^2+x+1} - \sqrt{x^2-x+3}}{x-1} = \frac{2}{\sqrt{x^2+x+1} + \sqrt{x^2-x+3}} \rightarrow \frac{1}{\sqrt{3}}.$$

$$(10) \frac{\sqrt{4x^2+2x+3} - \sqrt{4x^2+x+4}}{x-1} = \frac{1}{\sqrt{4x^2+2x+3} + \sqrt{4x^2+x+4}} \rightarrow \frac{1}{6}.$$