微分積分学 II 演習問題 1

問題 1. 以下の関数の原始関数を求めよ.

(1) $\dfrac{1}{x^2}$　　(2) $\dfrac{1}{\sqrt{x}}$

(3) $\dfrac{1}{\sqrt[3]{x^2}}$　　(4) $e^x + \cos x - \sin x$

(5) $\dfrac{1}{\cos^2 x}$　　(6) $\dfrac{x}{\cos^2 x^2}$

(7) $\dfrac{\cos x}{\sin x}$　　(8) $(2x+1)e^{x^2+x}$

(9) $e^x \sin e^x$　　(10) $\dfrac{\cos \log x}{x}$

(11) $\dfrac{1}{\sqrt{1-x}}$　　(12) $e^{\sin x} \cos x$

(13) $\dfrac{\sin x \cos x}{1+\sin^2 x}$　　(14) $\dfrac{e^x}{1+e^x}$

(15) $\dfrac{1}{x^2+2x+3}$　　(16) $\dfrac{1}{\sqrt{4-x^2}}$

(17) $\dfrac{\log x}{x^2}$　　(18) xe^x

(19) $x \log x$　　(20) $x \sin x$

問題 2. 変数変換 $x = \frac{z^2-1}{2z}$ を用い, 以下の原始関数を求めよ.

(1) $\displaystyle\int \frac{1}{\sqrt{x^2+1}} dx$　　(2) $\displaystyle\int \sqrt{x^2+1} dx$

問題 3. 以下の有理関数の原始関数を求めよ.

(1) $\dfrac{2}{x^2-1}$　　(2) $\dfrac{3x}{x^2-x-2}$

(3) $\dfrac{3x+3}{x^2+x-2}$　　(4) $\dfrac{5x+1}{x^2+x-2}$

(5) $\dfrac{4x+1}{x^2-x-2}$　　(6) $\dfrac{x+4}{x^2+2x+1}$

(7) $\dfrac{2x+5}{x^2+4x+4}$　　(8) $\dfrac{2x-3}{8x^3-12x^2+6x-1}$

(9) $\dfrac{x^2-3x}{x^3-x^2-x+1}$　　(10) $\dfrac{2x^2-x+8}{x^3+3x-2}$

(11) $\dfrac{x^2+2}{2x^3-9x^2+12x-4}$　　(12) $\dfrac{x^2+2x-1}{x^3+x^2+x+1}$

(13) $\dfrac{x+1}{x^3-x^2+x-1}$　　(14) $\dfrac{x-2}{x^3+x^2+2x+2}$

(15) $\dfrac{x^2+x}{x^3-x^2+x-1}$ (16) $\dfrac{2x^2-x}{x^3+2x^2+x+2}$

(17) $\dfrac{4x^2-x}{4x^3+4x^2+x+1}$ (18) $\dfrac{x^2+2x-5}{x^3-2x^2-x+2}$

(19) $\dfrac{2x^2+1}{x^3-3x+2}$ (20) $\dfrac{x^2-x+1}{x^3-3x^2+3x-1}$

問題 4. 以下の定積分の値を求めよ.

(1) $\displaystyle\int_0^1 (2x+1)dx$ (2) $\displaystyle\int_0^1 (x^2+x)dx$

(3) $\displaystyle\int_0^1 (4x^3-3x^2+2x-1)dx$ (4) $\displaystyle\int_0^1 \sqrt{x}dx$

(5) $\displaystyle\int_0^{\pi/3} \tan x dx$ (6) $\displaystyle\int_0^{\pi} \cos^2 x dx$

(7) $\displaystyle\int_{-1}^1 x^2 e^{x^3} dx$ (8) $\displaystyle\int_0^{\sqrt{3}} \frac{x}{x^2+1} dx$

(9) $\displaystyle\int_0^{2\sqrt{2}} \frac{x}{\sqrt{x^2+1}} dx$ (10) $\displaystyle\int_0^1 \frac{4x^3}{x^4+1} dx$

(11) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin x} dx$ (12) $\displaystyle\int_e^{e^2} \frac{1}{x\log x} dx$

(13) $\displaystyle\int_1^e \frac{\log x}{x} dx$ (14) $\displaystyle\int_0^1 \frac{1}{x^2+1} dx$

(15) $\displaystyle\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}} dx$ (16) $\displaystyle\int_1^e \log x dx$

(17) $\displaystyle\int_0^1 (x^2-1)e^x dx = -1$ (18) $\displaystyle\int_0^{\pi} x\cos x dx$

(19) $\displaystyle\int_0^1 \frac{3}{2x^2+5x+2} dx$ (20) $\displaystyle\int_0^1 \frac{8}{3x^2+10x+3} dx$

問題 5. 以下の I_n が満たす漸化式を求めよ.

(1) $I_n = \displaystyle\int x^n \sin x dx$ (2) $I_n = \displaystyle\int x^n \cos x dx$ (3) $I_n = \displaystyle\int \frac{1}{\cos^n x} dx$

問題 6. m, n を整数とするとき, 以下の定積分の値を求めよ.

$$\int_0^{\pi} \sin mx \sin nx dx$$

解答

問題 1.

(1) $-\dfrac{1}{x}$ (2) $2\sqrt{x}$

(3) $3\sqrt[3]{x}$ (4) $e^x + \sin x + \cos x$

(5) $\tan x$ (6) $\dfrac{1}{2}\tan x^2$

(7) $\log \sin x$ (8) e^{x^2+x}

(9) $-\cos e^x$ (10) $\sin \log x$

(11) $-2\sqrt{1-x}$ (12) $e^{\sin x}$

(13) $\dfrac{1}{2}\log(1+\sin^2 x)$ (14) $\log(1+e^x)$

(15) $\dfrac{1}{\sqrt{2}}\tan^{-1}\dfrac{x+1}{\sqrt{2}}$ (16) $\sin^{-1}\dfrac{x}{2}$

(17) $-\dfrac{1}{x}\log x - \dfrac{1}{x}$ (18) $(x-1)e^x$

(19) $\dfrac{x^2}{2}\log x - \dfrac{x^2}{4}$ (20) $-x\cos x + \sin x$

問題 2.

(1) $\log(x+\sqrt{x^2+1})$ (2) $\dfrac{1}{2}\left(x\sqrt{x^2+1}+\log(x+\sqrt{x^2+1})\right)$

問題 3.

(1) $\displaystyle\int\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx = \log(x-1)-\log(x+1)$

(2) $\displaystyle\int\left(\frac{1}{x+1}+\frac{2}{x-2}\right)dx = \log(x+1)+2\log(x-2)$

(3) $\displaystyle\int\left(\frac{2}{x-1}+\frac{1}{x+2}\right)dx = 2\log(x-1)+\log(x+2)$

(4) $\displaystyle\int\left(\frac{2}{x-1}+\frac{3}{x+2}\right)dx = 2\log(x-1)+3\log(x+2)$

(5) $\displaystyle\int\left(\frac{1}{x+1}+\frac{3}{x-2}\right)dx = \log(x+1)+3\log(x-2)$

(6) $\displaystyle\int\left(\frac{1}{x+1}+\frac{3}{(x+1)^2}\right)dx = \log(x+1)-\frac{3}{x+1}$

(7) $\displaystyle\int\left(\frac{2}{x+2}+\frac{1}{(x+2)^2}\right)dx = 2\log(x+2)-\frac{1}{x+2}$

(8) $\displaystyle\int\left(\frac{1}{2x-1}-\frac{2}{(2x-1)^2}\right)dx = \frac{1}{2}\log(2x-1)+\frac{1}{2x-1}$

(9) $\displaystyle\int\left(\frac{1}{x+1}-\frac{1}{(x-1)^2}\right)dx = \log(x+1)+\frac{1}{x-1}$

(10) $\displaystyle\int\left(\frac{2}{x+2}+\frac{3}{(x-1)^2}\right)dx = 2\log(x+1)-\frac{3}{x-1}$

(11) $\displaystyle\int\left(\frac{1}{2x-1}+\frac{2}{(x-2)^2}\right)dx=\frac{1}{2}\log(2x-1)-\frac{2}{x-2}$

(12) $\displaystyle\int\left(\frac{-1}{x+1}+\frac{2x}{x^2+1}\right)dx=-\log(x+1)+\log(x^2+1)$

(13) $\displaystyle\int\left(\frac{1}{x-1}-\frac{x}{x^2+1}\right)dx=\log(x-1)-\frac{1}{2}\log(x^2+1)$

(14) $\displaystyle\int\left(\frac{-1}{x+1}+\frac{x}{x^2+2}\right)dx=-\log(x+1)+\frac{1}{2}\log(x^2+2)$

(15) $\displaystyle\int\left(\frac{1}{x-1}+\frac{1}{x^2+1}\right)dx=\log(x-1)+\tan^{-1}x$

(16) $\displaystyle\int\left(\frac{2}{x+2}-\frac{1}{x^2+1}\right)dx=2\log(x+2)-\tan^{-1}x$

(17) $\displaystyle\int\left(\frac{1}{x+1}-\frac{1}{4x^2+1}\right)dx=\log(x+1)-\frac{1}{2}\tan^{-1}2x$

(18) $\displaystyle\int\left(\frac{1}{x-1}-\frac{1}{x+1}+\frac{1}{x-2}\right)dx=\log(x-1)-\log(x+1)+\log(x-2)$

(19) $\displaystyle\int\left(\frac{1}{x+2}+\frac{1}{x-1}+\frac{1}{(x-1)^2}\right)dx=\log(x+2)+\log(x-1)-\frac{1}{x-1}$

(20) $\displaystyle\int\left(\frac{1}{x-1}+\frac{1}{(x-1)^2}+\frac{1}{(x-1)^3}\right)dx=\log(x-1)-\frac{1}{x-1}-\frac{1}{2(x-1)^2}$

問題 4.

(1)	2	(2)	$\frac{5}{6}$	(3)	0	(4)	$\frac{2}{3}$	(5)	$\log 2$
(6)	$\frac{\pi}{2}$	(7)	$\frac{1}{3}(e-e^{-1})$	(8)	$\log 2$	(9)	2	(10)	$\log 2$
(11)	$\log 2$	(12)	$\log 2$	(13)	$\frac{1}{2}$	(14)	$\frac{\pi}{4}$	(15)	$\frac{\pi}{4}$
(16)	1	(17)	-1	(18)	-2	(19)	$\log 2$	(20)	$\log 3$

問題 5.

(1) $I_n=-x^n\cos x+nx^{n-1}\sin x-n(n-1)I_{n-2}$

(2) $I_n=x^n\sin x+nx^{n-1}\cos x-n(n-1)I_{n-2}$

(3) $I_{n+2}=\dfrac{1}{n+1}\dfrac{\sin x}{\cos^{n+1}x}+\dfrac{n}{n+1}I_n$

問題 6.

$$\int_0^{\pi}\sin mx\sin nxdx=\begin{cases}\frac{\pi}{2} & (m=n\neq 0)\\ -\frac{\pi}{2} & (m=-n\neq 0)\\ 0 & (\text{その他})\end{cases}$$