

微分積分学 II 演習問題 3

問題 1. 以下の関数  $f(x, y)$  を領域  $D$  上で積分した値  $\iint_D f(x, y) dx dy$  を求めよ.

- [1]  $f(x, y) = xy, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}.$
- [2]  $f(x, y) = x^2 y, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}.$
- [3]  $f(x, y) = x^2 + y^2, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}.$
- [4]  $f(x, y) = x^3 + x^2 y + xy^2 + y^3, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}.$
- [5]  $f(x, y) = \sqrt{xy}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$
- [6]  $f(x, y) = x^{\frac{1}{2}} y + x^{\frac{3}{2}}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$
- [7]  $f(x, y) = e^{x+y}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$
- [8]  $f(x, y) = xe^{xy}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$
- [9]  $f(x, y) = \cos(x + y), \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2\}.$
- [10]  $f(x, y) = x \cos(xy), \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq \pi\}.$

問題 2. 以下の関数  $f(x, y)$  を領域  $D$  上で積分した値  $\iint_D f(x, y) dx dy$  を求めよ.

- [1]  $f(x, y) = xy, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, x \leq y \leq 2\}.$
- [2]  $f(x, y) = x^2 y, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq 1, y \leq x \leq 1\}.$
- [3]  $f(x, y) = x^2 + y^2, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 1 - x \leq y \leq 1\}.$
- [4]  $f(x, y) = \frac{y+1}{x}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq y+1, 0 \leq y \leq 1\}.$
- [5]  $f(x, y) = \frac{1}{xy}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq 2, 1 \leq y \leq x+1\}.$
- [6]  $f(x, y) = x^{\frac{1}{2}} y + x^{\frac{3}{2}}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq y^2, 0 \leq y \leq 1\}.$
- [7]  $f(x, y) = e^{x+y}, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq 2, 0 \leq y \leq \log x\}.$
- [8]  $f(x, y) = xe^y, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq x^2\}.$
- [9]  $f(x, y) = x \sin y, \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq \pi, 0 \leq y \leq x\}.$
- [10]  $f(x, y) = y \cos \pi(y^2 - x), \quad D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq y^2, 0 \leq y \leq 1\}.$

以上.

解答

問題 1.

[1] 9	[2] 12	[3] 26	[4] 27	[5] $\frac{4}{9}$
[6] $\frac{11}{15}$	[7] $(e-1)^2$	[8] $e-2$	[9] 2	[10] $\frac{2}{\pi}$

問題 2.

[1] 2	[2] $\frac{1}{10}$	[3] $\frac{1}{2}$	[4] $\frac{1}{2}$	[5] $\frac{1}{2}$
[6] $\frac{1}{5}$	[7] $e$	[8] $\frac{e-2}{2}$	[9] $\pi$	[10] $\frac{1}{\pi^2}$