

線形代数学 II 演習問題 (2013 年 11 月 18 日)

問題 1. 以下では, 行列  $F \in M(m, n, \mathbb{R})$  に対し, 左からの掛算が定める線形写像  $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$  を考える. (すなわち,  $v \in \mathbb{R}^n$  に対して,  $f(v) = Fv$  とする.) ベクトル空間  $\mathbb{R}^n$  と  $\mathbb{R}^m$  の以下で与えられる基底  $\{v_1, \dots, v_n\}, \{w_1, \dots, w_m\}$  について,  $f$  の行列表示  $A_f$  を求めよ.

$$[1] F = \begin{pmatrix} 3 & -1 \\ 1 & -1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}.$$

$$[2] F = \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}.$$

$$[3] F = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}.$$

$$[4] F = \begin{pmatrix} 0 & 3 \\ 1 & -1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

$$[5] F = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

$$[6] F = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}.$$

$$[7] F = \begin{pmatrix} 1 & 1 & 3 \\ 3 & -1 & -1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}, \quad \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}.$$

$$[8] F = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}.$$

$$[9] F = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 2 & 1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

$$[10] F = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 1 & -2 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}.$$

$$[11] F = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}.$$

$$[12] F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

$$[13] F = \begin{pmatrix} 3 & 0 \\ -3 & 0 \\ 2 & 1 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}.$$

$$[14] F = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 2 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

$$[15] F = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}.$$

$$\left\{ v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}, \left\{ w_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

[16]

$$F = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \left\{ v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\},$$

$$\left\{ w_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

[17]

$$F = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \left\{ v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\},$$

$$\left\{ w_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

[18]

$$F = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}, \quad \left\{ v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\},$$

$$\left\{ w_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

[19]

$$F = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix}, \quad \left\{ v_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right\},$$

$$\left\{ w_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, w_3 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \right\}.$$

[20]

$$F = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}, \quad \left\{ v_1 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\},$$

$$\left\{ w_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

以上.

解答  
問題 1.

[1]  $\begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$

[2]  $\begin{pmatrix} -2 & 0 \\ 0 & 4 \end{pmatrix}$

[3]  $\begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix}$

[4]  $\frac{1}{2} \begin{pmatrix} 3 & -1 \\ 3 & -5 \end{pmatrix}$

[5]  $\frac{1}{2} \begin{pmatrix} 11 & 7 \\ -1 & 1 \end{pmatrix}$

[6]  $\begin{pmatrix} 2 & 3 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

[7]  $\frac{1}{2} \begin{pmatrix} 2 & 1 & 3 \\ 0 & -3 & -1 \end{pmatrix}$

[8]  $\begin{pmatrix} 1 & 3 & 6 \\ 2 & 2 & 0 \end{pmatrix}$

[9]  $\frac{1}{2} \begin{pmatrix} 0 & 0 & 2 \\ 1 & 2 & -2 \end{pmatrix}$

[10]  $\frac{1}{2} \begin{pmatrix} 1 & 3 & 5 \\ -5 & -1 & -3 \end{pmatrix}$

[11]  $\begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}$

[12]  $\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix}$

[13]  $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$

[14]  $\begin{pmatrix} 9 & -3 \\ -9 & 3 \\ 3 & -1 \end{pmatrix}$

[15]  $\frac{1}{2} \begin{pmatrix} -1 & 1 \\ 3 & 3 \\ 5 & 1 \end{pmatrix}$

[16]  $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

[17]  $\begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

[18]  $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -1 & 2 \end{pmatrix}$

[19]  $\begin{pmatrix} 0 & 1 & 4 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$

[20]  $\frac{1}{2} \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 3 & 1 & -3 \end{pmatrix}$