線形代数学 I 演習問題 (2014 年 4 月 14 日)

問題 1. 以下のベクトルを計算せよ.

[1] $2\begin{pmatrix}0\\1\\1\end{pmatrix}+3\begin{pmatrix}2\\-1\\1\end{pmatrix}$,

[2] $3\begin{pmatrix}1\\1\\2\end{pmatrix}-4\begin{pmatrix}1\\2\\2\end{pmatrix}$,

[3] $2\begin{pmatrix}3\\-2\\1\end{pmatrix}-\begin{pmatrix}2\\3\\3\end{pmatrix}$,

[4] $2\begin{pmatrix}2\\1\\-1\end{pmatrix}+\begin{pmatrix}3\\3\\1\end{pmatrix}$,

[5] $2\begin{pmatrix}3\\5\\-1\end{pmatrix}+\begin{pmatrix}-6\\2\\4\end{pmatrix}$,

[6] $3\begin{pmatrix}1\\1\\0\end{pmatrix}-3\begin{pmatrix}0\\1\\1\end{pmatrix}$,

[7] $2\begin{pmatrix}1\\0\\-1\end{pmatrix}+5\begin{pmatrix}-1\\0\\1\end{pmatrix}$,

[8] $-\begin{pmatrix}3\\1\\-1\end{pmatrix}+2\begin{pmatrix}2\\3\\3\end{pmatrix}$,

[9] $\begin{pmatrix}-1\\5\\2\end{pmatrix}+4\begin{pmatrix}1\\-1\\-1\end{pmatrix}$,

[10] $3\begin{pmatrix}-3\\2\\1\end{pmatrix}+3\begin{pmatrix}3\\-2\\-2\end{pmatrix}$,

[11] $3\begin{pmatrix}4\\-1\\2\end{pmatrix}-\begin{pmatrix}2\\0\\0\end{pmatrix}+2\begin{pmatrix}-5\\2\\-1\end{pmatrix}$,

[12] $\begin{pmatrix}2\\1\\-1\end{pmatrix}-\begin{pmatrix}1\\1\\1\end{pmatrix}+4\begin{pmatrix}1\\-1\\-1\end{pmatrix}$,

[13] $2\begin{pmatrix}0\\1\\1\end{pmatrix}+\begin{pmatrix}1\\0\\-2\end{pmatrix}+\begin{pmatrix}-1\\1\\3\end{pmatrix}$,

[14] $\begin{pmatrix}2\\0\\1\end{pmatrix}-\begin{pmatrix}1\\-1\\1\end{pmatrix}+\begin{pmatrix}0\\4\\2\end{pmatrix}$,

[15] $\begin{pmatrix}1\\3\\2\end{pmatrix}-2\begin{pmatrix}3\\5\\-1\end{pmatrix}-\begin{pmatrix}-1\\-1\\2\end{pmatrix}$,

[16] $\begin{pmatrix}0\\2\\2\end{pmatrix}+2\begin{pmatrix}-3\\3\\-1\end{pmatrix}+\begin{pmatrix}-1\\1\\0\end{pmatrix}$,

[17] $2\begin{pmatrix}0\\0\\-3\end{pmatrix}-2\begin{pmatrix}2\\3\\3\end{pmatrix}+3\begin{pmatrix}1\\1\\0\end{pmatrix}$,

[18] $2\begin{pmatrix}2\\-1\\5\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}+2\begin{pmatrix}1\\-1\\0\end{pmatrix}$,

[19] $2\begin{pmatrix}-3\\1\\2\end{pmatrix}-\begin{pmatrix}1\\0\\-1\end{pmatrix}+\begin{pmatrix}1\\1\\-1\end{pmatrix}$,

[20] $\begin{pmatrix}1\\0\\2\end{pmatrix}-2\begin{pmatrix}2\\1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\\-1\end{pmatrix}$,

[21] $3\begin{pmatrix}-1\\1\\0\end{pmatrix}-\begin{pmatrix}2\\1\\-1\end{pmatrix}+\begin{pmatrix}1\\1\\-2\end{pmatrix}$,

[22] $\begin{pmatrix}0\\1\\1\end{pmatrix}-2\begin{pmatrix}2\\1\\1\end{pmatrix}+\begin{pmatrix}2\\-1\\-1\end{pmatrix}$,

[23] $-\begin{pmatrix}1\\1\\1\end{pmatrix}+2\begin{pmatrix}3\\4\\-1\end{pmatrix}-\begin{pmatrix}-1\\0\\3\end{pmatrix}$,

[24] $\begin{pmatrix}2\\2\\3\end{pmatrix}+2\begin{pmatrix}-3\\3\\-2\end{pmatrix}+\begin{pmatrix}1\\1\\-1\end{pmatrix}$.

問題 2. 以下が成立するような $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ を求めよ.

$$[1] \quad \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix},$$

$$[2] \quad \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \end{pmatrix},$$

$$[3] \quad \alpha \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix},$$

$$[4] \quad \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix},$$

$$[5] \quad \alpha \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} -6 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 17 \end{pmatrix},$$

$$[6] \quad \alpha \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$[7] \quad \alpha \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},$$

$$[8] \quad \alpha \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

$$[9] \quad \alpha \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix},$$

$$[10] \quad \alpha \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix},$$

$$[11] \quad \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -1 \\ 7 \end{pmatrix},$$

$$[12] \quad \alpha \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ 5 \\ -3 \end{pmatrix},$$

$$[13] \quad \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 1 \end{pmatrix},$$

$$[14] \quad \alpha \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -4 \\ -3 \end{pmatrix}.$$

以上.

解答

問題 1.

$$[1]\ \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix},\quad [2]\ \begin{pmatrix} -1 \\ -5 \\ -2 \end{pmatrix},\quad [3]\ \begin{pmatrix} 4 \\ -7 \\ -1 \end{pmatrix},\quad [4]\ \begin{pmatrix} 7 \\ 5 \\ -1 \end{pmatrix},$$

$$[5]\ \begin{pmatrix} 0 \\ 12 \\ 2 \end{pmatrix},\quad [6]\ \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix},\quad [7]\ \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix},\quad [8]\ \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix},$$

$$[9]\ \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix},\quad [10]\ \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix},\quad [11]\ \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix},\quad [12]\ \begin{pmatrix} 5 \\ -4 \\ -6 \end{pmatrix},$$

$$[13]\ \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix},\quad [14]\ \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix},\quad [15]\ \begin{pmatrix} -4 \\ -6 \\ 2 \end{pmatrix},\quad [16]\ \begin{pmatrix} -7 \\ 9 \\ 0 \end{pmatrix},$$

$$[17]\ \begin{pmatrix} -1 \\ -3 \\ -12 \end{pmatrix},\quad [18]\ \begin{pmatrix} 5 \\ -5 \\ 6 \end{pmatrix},\quad [19]\ \begin{pmatrix} -6 \\ 3 \\ 4 \end{pmatrix},\quad [20]\ \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix},$$

$$[21]\ \begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix},\quad [22]\ \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix},\quad [23]\ \begin{pmatrix} 6 \\ 7 \\ -6 \end{pmatrix},\quad [24]\ \begin{pmatrix} -3 \\ 9 \\ -2 \end{pmatrix}.$$

問題 2.

[1] $\alpha = 2, \beta = 1.$

[2] $\alpha = 5, \beta = 2.$

[3] $\alpha = 2, \beta = 3.$

[4] $\alpha = -2, \beta = 2.$

[5] $\alpha = 3, \beta = 1.$

[6] $\alpha = 1, \beta = 1, \gamma = 1.$

[7] $\alpha = 1, \beta = -1, \gamma = -1.$

[8] $\alpha = 1, \beta = 1, \gamma = 0.$

[9] $\alpha = -1, \beta = -1, \gamma = 1.$

[10] $\alpha = 3, \beta = -2, \gamma = -1.$

[11] $\alpha = 1, \beta = -2, \gamma = 3, \delta = -4.$

[12] $\alpha = 3, \beta = 1, \gamma = -4, \delta = -1.$

[13] $\alpha = 1, \beta = 2, \gamma = -2, \delta = 1.$

[14] $\alpha = 2, \beta = 1, \gamma = 1, \delta = -3.$