

線形代数学 I 演習問題 (2014 年 5 月 19 日)

問題. 以下の連立一次方程式の一般解を求めよ.

$$\begin{array}{l}
 [1] \left\{ \begin{array}{l} -x_1 + x_2 - 2x_3 - x_4 = -1 \\ 3x_1 - 3x_2 + 6x_3 - 3x_4 = 9 \\ -5x_1 + 5x_2 - 10x_3 - 5x_4 = -5 \\ 2x_1 - 2x_2 + 4x_3 + 2x_4 = 2 \end{array} \right. \\
 [2] \left\{ \begin{array}{l} -2x_1 + 2x_2 - 4x_3 - 2x_4 = 2 \\ 3x_1 - 3x_2 + 6x_3 + 3x_4 = -3 \\ -x_1 + x_2 - 2x_3 - x_4 = 1 \\ x_1 - x_2 + 2x_3 + x_4 = -1 \end{array} \right. \\
 [3] \left\{ \begin{array}{l} 2x_1 + 7x_2 + 4x_3 - 3x_4 = 2 \\ -2x_1 + \quad -4x_3 - 4x_4 = -2 \\ \quad -2x_2 \quad + 2x_4 = 0 \\ 2x_1 + 4x_2 + 4x_3 \quad = 2 \end{array} \right. \\
 [4] \left\{ \begin{array}{l} 2x_1 + x_2 - 2x_3 - 2x_4 = 1 \\ 2x_1 + 2x_2 + 6x_3 + 5x_4 = 1 \\ x_1 + x_2 + 5x_3 + 4x_4 = 1 \\ x_1 + x_2 + 6x_3 + 5x_4 = 1 \end{array} \right. \\
 [5] \left\{ \begin{array}{l} \quad -3x_2 \quad + 3x_4 = 3 \\ -x_1 + 2x_2 - 4x_3 - 5x_4 = 0 \\ -3x_1 + 6x_2 \quad - 9x_4 = 0 \\ -2x_1 + 4x_2 \quad - 6x_4 = 0 \end{array} \right. \\
 [6] \left\{ \begin{array}{l} 2x_1 + 5x_2 + 4x_3 - x_4 = 1 \\ 2x_1 - x_2 + 4x_3 + 5x_4 = 7 \\ 5x_1 + 11x_2 + 10x_3 - x_4 = 4 \\ -x_1 - 4x_2 - 2x_3 + 2x_4 = 1 \end{array} \right. \\
 [7] \left\{ \begin{array}{l} x_1 + 2x_2 + 2x_3 + x_4 = 1 \\ 5x_1 + 8x_2 + 10x_3 + 2x_4 = -3 \\ x_1 + 2x_2 + 2x_3 \quad = -1 \\ x_1 \quad + 2x_3 + 2x_4 = 1 \end{array} \right. \\
 [8] \left\{ \begin{array}{l} \quad -5x_2 - 10x_3 - 6x_4 = 1 \\ x_1 + 4x_2 + 4x_3 + 2x_4 = 0 \\ \quad x_2 + 2x_3 + x_4 = 0 \\ \quad x_2 + 7x_3 + 4x_4 = 0 \end{array} \right. \\
 [9] \left\{ \begin{array}{l} x_1 \quad + 2x_3 + 2x_4 = 1 \\ x_1 - 2x_2 + 2x_3 + 4x_4 = -1 \\ -3x_1 + 2x_2 - 6x_3 - 8x_4 = -1 \\ \quad 3x_2 \quad - 3x_4 = 3 \end{array} \right. \\
 [10] \left\{ \begin{array}{l} 2x_1 + x_2 + 4x_3 + 3x_4 = 3 \\ x_1 + 5x_2 + 2x_3 - 3x_4 = 6 \\ -4x_1 + x_2 - 6x_3 - 8x_4 = -2 \\ x_1 - 2x_2 - 2x_3 + 2x_4 = -3 \end{array} \right. \\
 [11] \left\{ \begin{array}{l} 3x_1 + 6x_2 + 6x_3 \quad = 3 \\ -2x_1 + 4x_2 - 4x_3 - 8x_4 = -2 \\ x_1 + x_2 + 2x_3 + x_4 = 1 \\ 2x_2 \quad - 2x_4 = 0 \end{array} \right. \\
 [12] \left\{ \begin{array}{l} -3x_1 + x_2 + 3x_3 - 2x_4 = 1 \\ \quad -x_2 - 3x_3 + 3x_4 = 4 \\ x_1 - x_2 - 4x_3 + 4x_4 = 4 \\ \quad -x_3 + x_4 = 1 \end{array} \right. \\
 [13] \left\{ \begin{array}{l} 4x_1 + 6x_2 + 8x_3 + 2x_4 = 0 \\ -3x_1 - 7x_2 - 6x_3 + x_4 = 0 \\ \quad -x_2 \quad + x_4 = 0 \\ -2x_1 + 2x_2 - 4x_3 - 6x_4 = 0 \end{array} \right. \\
 [14] \left\{ \begin{array}{l} 2x_1 + 3x_2 + 4x_3 + x_4 = -1 \\ -3x_1 - 5x_2 - 6x_3 + x_4 = 4 \\ \quad -x_2 \quad + x_4 = 1 \\ -2x_1 + 2x_2 - 4x_3 - 6x_4 = -4 \end{array} \right. \\
 [15] \left\{ \begin{array}{l} -2x_1 - 3x_2 - 4x_3 + x_4 = -4 \\ 3x_1 + 3x_2 + 6x_3 \quad = 6 \\ \quad 2x_2 \quad - 2x_4 = 0 \\ 3x_1 - 3x_2 + 6x_3 + 6x_4 = 7 \end{array} \right. \\
 [16] \left\{ \begin{array}{l} -x_1 - 8x_2 - 4x_3 - 7x_4 = 2 \\ \quad 2x_3 + x_4 = 1 \\ 2x_2 + 3x_3 + 3x_4 = 1 \\ 3x_2 + 5x_3 + 5x_4 = 2 \end{array} \right. \\
 [17] \left\{ \begin{array}{l} 3x_1 - 3x_2 \quad + 3x_4 = 3 \\ x_1 - x_2 - x_3 + x_4 = 2 \\ 2x_1 - 2x_2 \quad + 2x_4 = 2 \\ -x_1 + x_2 - 3x_3 - x_4 = 2 \end{array} \right. \\
 [18] \left\{ \begin{array}{l} x_1 + x_2 + x_3 - 6x_4 = 3 \\ 4x_1 - 6x_2 \quad - 4x_4 = 6 \\ 3x_1 - 5x_2 - x_3 + 2x_4 = 1 \\ 2x_1 - 3x_2 \quad - 2x_4 = 3 \end{array} \right.
 \end{array}$$

解答

問題 1. (解答例) 以下, a, b, c は任意の定数である.

$$\begin{array}{ll}
 [1] \quad \begin{cases} x_1 = a - 2b + 2, \\ x_2 = a, \\ x_3 = b, \\ x_4 = -1. \end{cases} & [2] \quad \begin{cases} x_1 = a - 2b - c - 1, \\ x_2 = a, \\ x_3 = b, \\ x_4 = c. \end{cases} \\
 [3] \quad \begin{cases} x_1 = -2a - 2b + 1, \\ x_2 = b, \\ x_3 = a, \\ x_4 = b. \end{cases} & [4] \quad \begin{cases} x_1 = 1, \\ x_2 = -1, \\ x_3 = 1, \\ x_4 = -1. \end{cases} \\
 [5] \quad \begin{cases} x_1 = -a - 2, \\ x_2 = a - 1, \\ x_3 = -a/2, \\ x_4 = a, \end{cases} & [6] \quad \begin{cases} x_1 = -2a - 2b + 3, \\ x_2 = b - 1, \\ x_3 = a, \\ x_4 = b. \end{cases} \\
 [7] \quad \begin{cases} x_1 = -2a - 3, \\ x_2 = 1, \\ x_3 = a, \\ x_4 = 2. \end{cases} & [8] \quad \begin{cases} x_1 = 2/5, \\ x_2 = -1/5, \\ x_3 = 3/5, \\ x_4 = -1. \end{cases} \\
 [9] \quad \begin{cases} x_1 = -2a - 2b + 1, \\ x_2 = b + 1, \\ x_3 = a, \\ x_4 = b. \end{cases} & [10] \quad \begin{cases} x_1 = -a, \\ x_2 = a + 1, \\ x_3 = (-a + 1)/2, \\ x_4 = a. \end{cases} \\
 [11] \quad \begin{cases} x_1 = -2a - 2b + 1, \\ x_2 = b, \\ x_3 = a, \\ x_4 = b. \end{cases} & [12] \quad \begin{cases} x_1 = -1, \\ x_2 = -1, \\ x_3 = 1, \\ x_4 = 2. \end{cases} \\
 [13] \quad \begin{cases} x_1 = -2a - 2b, \\ x_2 = b, \\ x_3 = a, \\ x_4 = b. \end{cases} & [14] \quad \begin{cases} x_1 = -2a - 1, \\ x_2 = 0, \\ x_3 = a, \\ x_4 = 1. \end{cases} \\
 [15] \quad \text{解なし.} & [16] \quad \begin{cases} x_1 = -1, \\ x_2 = -1, \\ x_3 = 0, \\ x_4 = 1. \end{cases} \\
 [17] \quad \begin{cases} x_1 = a - b + 1, \\ x_2 = a, \\ x_3 = -1, \\ x_4 = b. \end{cases} & [18] \quad \begin{cases} x_1 = a, \\ x_2 = -1, \\ x_3 = 5a + 4, \\ x_4 = a. \end{cases}
 \end{array}$$