

**TOPICS IN K-THEORY OF OPERATOR ALGEBRAS:
DADARLAT-PENNIG'S GENERALIZATION OF DIXMIER-DOUADY
THEORY AND GROUP ACTIONS ON KIRCHBERG ALGEBRAS I, II**

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The K -theory of C^* -algebras is a direct generalization of topological K -theory of locally compact Hausdorff spaces. In the talk, I give an overview of recent topics on the K -theory of C^* -algebras related to algebraic topology.

The classical Dixmier-Douady theory describes the structure of continuous trace C^* -algebras in terms of the third cohomology of their primitive spectra, which can be rephrased as follows: the classifying space $B\text{Aut}(\mathbb{K})$ of the automorphism group of the C^* -algebra of compact operators \mathbb{K} on ℓ^2 is an Eilenberg-MacLane space of type $K(\mathbb{Z}, 3)$. In 1989, Rosenberg formulated twisted K -theory in full generality as the K -theory of a continuous trace C^* -algebra with its spectrum homeomorphic to a prescribed space and with a prescribed third cohomology class. Since then twisted K -theory has been extensively studied, partly because its relationship with string theory was revealed in the late '90s (see [4], [5]).

On the other hand, in the Elliott program of the classification of nuclear C^* -algebras, the importance of a certain class of C^* -algebras with very simple structure, now known as strongly self-absorbing C^* -algebras, had been recognized among the specialists long before their formal definition was introduced by Toms-Winter [6] in 2007. Recently, a surprising and unexpected relationship between them and algebraic topology was found by Dadarlat-Pennig [1], [2], [3], who showed that the Dixmier-Douady theory can be generalized to every strongly self-absorbing C^* -algebra A in that the classical Dixmier-Douady theory is for the trivial C^* -algebra \mathbb{C} , the complex numbers. More precisely, they showed that the classifying space $B\text{Aut}(A \otimes \mathbb{K})$ has an infinite loop space structure.

In the talk, I focus on the Dadarlat-Pennig theory, and related work of Hiroki Matui and myself on poly- \mathbb{Z} group actions on Kirchberg algebras.

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