Ideas and techniques in the theory of Lefschetz properties

Toshiaki Maeno

The Lefschetz properties are ring-theoretic abstraction of the Hard Lefschetz Theorem for the cohomology ring of compact Kaehler manifolds. They just deal with ranks of multiplication maps. However, it is often surprisingly difficult to show the Lefschetz property for a given algebra in purely algebraic manners. It is also interesting that we can sometimes get some combinatorial gifts by showing the Lefschetz property for an algebra. In the previous half of my talk, I will survey the motivations, basic problems and methods to show the Lefschetz properties. In the latter half, we examine some examples of algebras with the Lefschetz property in view of combinatorial applications.