Categories of operators over group operads and homology of algebras

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Algebraic structures are studied by many researchers; in particular, homotopy invariant ones are important in algebraic topology. One of the popular formalizations uses operads. The notion was first introduced by May based on Stasheff's idea of associahedra. He used it to prove the Recognition Theorem, which clarifies the relation between commutativity of products and iterated loop space structures.

Operads are by definition a family of objects in a symmetric monoidal category; e.g. sets, spaces, modules, chain complexes, and so on. In 1978, May and Thomason gave another formalism of operads; namely *categories of operators*. They are a kind of fibrations over a certain base category, so the notion is very convenient in homotopical context. In fact, Lurie developed the notion of ∞-*operads* based on the idea, and it is really popular in the recent study of higher algebras.

We note that, in the above, we use the terminology "operads" in the sense that they by definition have actions of symmetric groups in a way compatible with their structures. That is why they are sometimes called *symmetric operads*. On the other hand, in 2001, Wahl considered a class of non-symmetric operads which can play roles of symmetric groups in the definition of symmetric operads; such operads are later called *group operads* or *action operads*. For each group operad $\mathcal{G}$, we obtain the notion of $\mathcal{G}$-operads so that it agrees with that of symmetric operads in the case $\mathcal{G}$ is the operad of symmetric groups. Motivational examples are braided operads and ribbon operads.

The goal of the talk is to discuss an analogue of categories of operators for $\mathcal{G}$-operads for general group operads $\mathcal{G}$. The key is the construction of the base category $\mathbb{B}_{\mathcal{G}}$, which was obtained in the speaker's thesis. The plan is as follows.

(1) We first review the definition and basic facts on operads.

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(2) The definition of group operads are explained. In particular, it will be seen how braided and ribbon monoidal categories are described in terms of the operads of braid group and ribbon braid groups respectively.

(3) We construct the base category $\mathbb{B}_{\mathcal{G}}$. We also see that $\mathcal{G}$-operads give rise to fibrations over $\mathbb{B}_{\mathcal{G}}$. A brief explanation of the equivalence will be given.

(4) Finally, algebras in $\mathcal{G}$-monoidal categories are discussed. We introduce $\mathcal{G}$-bar resolutions of them to define a sort of homologies. In particular, Hochschild homology is defined on algebras in braided monoidal categories.