

ON THE HIERARCHY INTERPOLATING RATIONALITY AND RULEDNESS, AND ITS GENERALIZATIONS

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The rationality problem is of fundamental importance in algebraic geometry, highlighted by the classical Lüroth problem.

In these two talks, I shall report my effort to study this problem from much more fine point of view of the naive hierarchy interpolating rationality and ruledness.

Amongst of all, I shall focus upon two particular kinds of the rationality problem to be hierarchized. The first is the Noether problem of finite groups for the field of complex numbers, and the second is the rationality problem of complex hypersurfaces.

For the rationality problem of complex hypersurfaces, I shall upgrade the non rationality results for very general complex hypersurfaces of Kollár, Totaro, Schreieder, and Nicaise-Ottem, from my hierarchical point of view.

On the other hand, the Noether problem of finite groups for the field of complex numbers essentially asks the stable rationality of the classifying space of finite groups. To upgrade previous works on this problem, I need my own local uniformization theorem (which is a local analogue of resolution of singularities) because classifying spaces are studied as an ind object of non proper scheme. In fact, my local uniformization theorem is especially useful to apply Morel's theory of unramified sheaves, which was developed to study \mathbb{A}^1 -algebraic topology (i.e. motivic homotopy theory), and this leads to some nice functoriality theorem for SBNR (i.e. stably birationalized unramified) sheaves, which enables me to upgrade previous works on the Noether problem of finite groups for the field of complex numbers for free.

If time permits, I shall also mention related hierarchical results of my own, like one on the counter-examples to the integral Hodge conjecture.

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