## Spaces of non-resultant systems of bounded multiplicity and homotopy stability

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For positive integers $m, n, d \geq 1$ with $(m, n) \neq (1, 1)$ and a field $\mathbb{F}$ with its algebraic closure $\overline{\mathbb{F}}$, let $\mathrm{Poly}_n^{d,m}(\mathbb{F})$ denote the space of all $m$-tuples $(f_1(z), \cdots, f_m(z)) \in \mathbb{F}[z]$ of $\mathbb{F}$-coefficients monic polynomials of the same degree $d$ such that polynomials $f_1(z), \cdots, f_m(z)$ have no common root in $\overline{\mathbb{F}}$ of multiplicity $\geq n$. The space $\mathrm{Poly}_n^{d,m}(\mathbb{F})$ was first considered and studied by Farb and Wolfson [5] for studying the homological densities of algebraic cycles in a manifold ([6]). This space can be also regarded as a generalization of spaces studied by Arnold, Vassiliev, Segal and others in different contexts (e.g. [2], [3], [8], [9], [16], [17]), and it is usually called *the space of non-resultant systems of bounded multiplicity with coefficients* $\mathbb{F}$. A. Kozlowski and the author [10] already studied about the homotopy type of these space for the case $\mathbb{F} = \mathbb{C}$. In this talk we shall mainly consider the homotopy type of the space $\mathrm{Poly}_n^{d,m}(\mathbb{F})$ for the case $\mathbb{F} = \mathbb{C}$ or $\mathbb{C}$ ([10], [14], [15]). Moreover, as one of generalizations, we may explain about the space $\mathrm{Poly}_n^{D,\Sigma}$ of non-resultant systems determined by a toric variety ([11], [13]).

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