The Steenrod algebra and its representations

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空間の代数的・幾何的モデルとその周辺

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Let $\mathcal{A}_p^*$ be the Steenrod algebra over the prime field of characteristic p. Since $\mathcal{A}_p^*$ has a structure of a cocommutative Hopf algebra, the dual $\mathcal{A}_{p*}$ of $\mathcal{A}_p^*$ is a commutative Hopf algebra. Hence we have an affine group scheme G_p represented by the dual Steenrod algebra $\mathcal{A}_{p*}$. On the other hand, J. Milnor defined a right $\mathcal{A}_p^*$-comodule structure on the mod p cohomology group $H^*(X)$ of a finite complex X from the left $\mathcal{A}_p^*$-module structure of $H^*(X)$ in his paper "The Steenrod algebra and its dual". This comodule structure is called "the Milnor coaction" which enable us to regard $H^*(X)$ as a representation of the affine group scheme G_p. The aim of this talk is to provide a foundation of the representations of G_p and give an interpretation of the Milnor coaction from the categorical viewpoint. In the first half of this talk, after reviewing the construction of the Milnor coaction, we collect basic facts on topological vector spaces which are needed to define the Milnor coaction on infinite dimensional A^*-modules and give a reformulation of the Milnor coaction. In the second half, we introduce the notions of "fibered category with products" and "fibered category with exponents" and develop a fundamental theory of representations of group objects in a category with finite products. Under this framework, we give a categorical definition of the Milnor coaction.