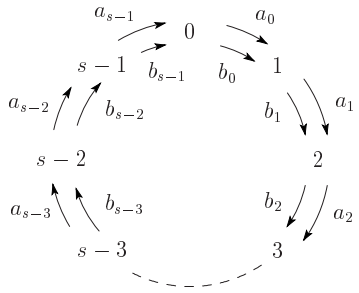
On Hochschild cohomology of a self-injective special biserial algebra obtained by a circular quiver with double arrows

Ayako Itaba (Tokyo University of Science)*

Let K be an algebraically closed field. For a positive integer s, let Γ_s be the following circular quiver with double arrows:



We set the elements $x = \sum_{i=0}^{s-1} a_i$ and $y = \sum_{i=0}^{s-1} b_i$ in the path algebra $K\Gamma_s$. We denote by I the ideal generated by x^2, $xy + yx$ and y^2. Then we define the bound quiver algebra $\Lambda_s = K\Gamma_s/I$ over K. This algebra Λ_s is a Koszul self-injective special biserial algebra ([I]).

We calculate the Hochschild cohomology group $\mathrm{HH}^n(\Lambda_s)$ of Λ_s for $n \geq 0$. Note that, for $s = 1, 2, 4$, the Hochschild cohomology of Λ_s is reserched in [XH], [ST] and [F], respectively. In the following, we assume that $s \geq 3$.

Theorem 1 ([I]). *Let $n = ms + r$ for integers $m \geq$ and $0 \leq r \leq s-1$. Then we have the dimension formula for the Hochschild cohomology groups of Λ_s as follows*:

$$\dim_K \mathrm{HH}^{ms+r}(\Lambda_s) = \begin{cases} ms+1 & \text{if } s \text{ even and } r = 0, \text{ if } m \text{ even and } r = 0, \text{ or} \\ & \quad \text{if } \mathrm{char}\, K = 2 \text{ and } r = 0, \\ 2ms+4 & \text{if } s \text{ even and } r = 1, \text{ if } m \text{ even and } r = 1, \text{or} \\ & \quad \text{if } \mathrm{char}\, K = 2 \text{ and } r = 1, \\ ms+3 & \text{if } s \text{ even and } r = 2, \text{ if } m \text{ even and } r = 2, \text{or} \\ & \quad \text{if } \mathrm{char}\, K = 2 \text{ and } r = 2, \\ 0 & \text{otherwise.} \end{cases}$$

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* e-mail: j1110701@ed.tus.ac.jp