Tilted algebras and configurations of self-injective algebras of Dynkin type

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This is a joint work with H. Asashiba, and is a generalization of H. Suzuki's Master thesis. Throughout this talk n is a positive integer and $\Bbbk$ is an algebraically closed field, and all algebras considered here are assumed to be basic, connected, finite-dimensional associative $\Bbbk$-algebras.

Let Δ be a Dynkin graph of type A, D, E with the set $\Delta_0 := \{1, \dots, n\}$ of vertices. We set $\mathbf{C}_n$ to be the set of configurations on the translation quiver $\mathbb{Z}\Delta$, and $\mathbf{T}_n$ to be the set of isoclasses of tilted algebras of type Δ. Then Bretscher, Läser and Riedtmann have given a bijection $c\colon \mathbf{T}_n \to \mathbf{C}_n$ in [1]. But the map c is not given in a direct way, it needs a long computation of a function on $\mathbb{Z}\Delta$. In this talk we give a direct formula for the map c. Let A be a tilted algebra of section type $\vec{\Delta}$, where $\vec{\Delta}$ is a quiver with underlying graph Δ. Then by identifying A with the $(0,0)$-entry of the repetitive category $\hat{A}$, the Auslander-Reiten quiver Γ_A of A is embedded into the stable Auslander-Reiten quiver ${}_s\Gamma_{\hat{A}} \cong \mathbb{Z}\vec{\Delta} = \mathbb{Z}\Delta$ of $\hat{A}$, and the configuration $\mathcal{C} := c(A)$ of $\mathbb{Z}\Delta$ computed in [1] is given by the vertices of $\mathbb{Z}\Delta$ corresponding to radicals of projective indecomposable $\hat{A}$-modules. Note that the configuration $\mathcal{C}$ has a period m_Δ, thus $\mathcal{C} = \tau^{m_\Delta \mathbb{Z}}\mathcal{F}$ for some subset $\mathcal{F}$ of $\mathcal{C}$. By $\mathcal{P} = \{(p(i), i) \mid i \in \Delta_0\}$ we denote the set of images of the projective vertices of Γ_A in $\mathbb{Z}\Delta$ and set

$$\mathbb{N}\mathcal{P} := \{(m, i) \in (\mathbb{Z}\Delta)_0 \mid p(i) \le m, i \in \Delta_0\}.$$

Since the mesh category $\Bbbk(\mathbb{Z}\Delta)$ is a Frobenius category, it has the Nakayama permutation ν on $(\mathbb{Z}\Delta)_0$, the precise formula of which is well-known. In the talk we will define a map $\nu'\colon \mathcal{P} \to \mathbb{N}\mathcal{P}$ using the supports of starting functions $\dim_\Bbbk \Bbbk(\mathbb{Z}\Delta)(x, \text{-})\colon \mathbb{N}\mathcal{P} \to \mathbb{Z}$ for $x \in \mathbb{N}\mathcal{P}$.

Lemma 1. *Let $x \in \mathcal{P}$ and P be the projective indecomposable A-module corresponding to x. Then $\nu' x$ corresponds to the simple module* $\operatorname{top} P$.

Lemma 2. *Assume that a vertex $x \in \mathbb{Z}\Delta$ corresponds to a simple $\hat{A}$-module S and let Q be the injective hull of S over $\hat{A}$. Then $\nu(x)$ corresponds to* $\operatorname{rad} Q$*, and hence $\nu(x) \in \mathcal{C}$.*

Proposition. *If $x \in \mathcal{P}$, then $\nu(\nu' x) \in \mathcal{C}$.*

Definition. We define a map $c_A\colon \mathcal{P} \to \mathcal{C}$ by $c_A(x) := \nu(\nu' x)$ for all $x \in \mathcal{P}$.

Then the map c_A enables us to compute the configuration $c(A) = \mathcal{C}$ as follows.

Theorem. *The map c_A is an injection, and we have $c(A) = \tau^{m_\Delta \mathbb{Z}} \operatorname{Im} c_A$.*

References

[1] O. Bretscher, Ch. Läser and Chr. Riedtmann, *Selfinjective and simply connected algebras*, manuscripta math. **36** (1981), 253–307.