CYCLIC HOMOLOGY OF TRUNCATED QUIVER ALGEBRAS AND NOTES ON THE NO LOOPS CONJECTURE FOR HOCHSCHILD HOMOLOGY

TOMOHIRO ITAGAKI

Abstract. In this paper, we give the dimension formula of the cyclic homology of truncated quiver algebras over a field of positive characteristic and we show the m-truncated cycles version of the no loops conjecture. This paper is based on joint work with Katsunori Sanada.

1. Introduction

Let Δ be a finite quiver and K a field. We fix a positive integer $m \geq 2$. The truncated quiver algebra is defined by $K\Delta/R_\Delta^m$ where R_Δ^m is the two-sided ideal of $K\Delta$ generated by the all paths of length m.

In [11], Sköldberg computes the Hochschild homology of a truncated quiver algebra A over a commutative ring using an explicit description of the minimal left A^e-projective resolution $\boldsymbol{P}$ of A. He also computes the Hochschild homology of quadratic monomial algebras. On the other hand, Cibils gives a useful projective resolution $\boldsymbol{Q}$ for more general algebras in [4]. If A is a K-algebra with a decomposition $A = E \oplus r$, where E is a separable subalgebra of A and r a two-sided ideal of A, then Cibils ([5]) gives the *E-normalized mixed complex*. Sköldberg [12] gives the chain maps between the left A^e-projective resolution given in [11] and $\boldsymbol{Q}$ above for a quadratic monomial algebra A, and he obtains the module structure of the cyclic homology by computing the E^2-term of a spectral sequence determined by the above mixed complex due to Cibils. In [1], Ames, Cagliero and Tirao give chain maps between the left A^e-projective resolutions $\boldsymbol{P}$ and $\boldsymbol{Q}$ of a truncated quiver algebra A over commutative ring.

In this paper, by means of these chain maps, we obtain the dimension formula of the cyclic homology of truncated quiver algebras over a field. On the other hand, by means of [10, Theorem 4.1.13], Taillefer [13] gives a dimension formula for the cyclic homology of truncated quiver algebras over a field of characteristic zero. Our result generalizes the formula into the case of the field of any characteristic.

Moreover, we have a result for the m-truncated cycles version of the no loops conjecture as an application of the chain map in [1] used for the computation of cyclic homology of truncated quiver algebras.

The no loops conjecture is that for a finite dimensional algebra its ordinary quiver has no loops if it has finite global dimension. In [3], it is shown that the 2-truncated cycles version of the no loops conjecture holds by means of truncated quiver algebras, and the m-truncated cycles version of one is conjectured. We show that the m-truncated cycles version of the no loops conjecture holds for a class of bound quiver algebras over an algebraically closed field as an application of the chain map from Cibils' projective resolution (cf. [4]) to Sköldberg's projective resolution given in [1].

2. Preliminaries

Let Δ be a finite quiver and $m(\geq 2)$ a positive integer. For $\alpha \in \Delta_1$, its source and target are denoted by $s(\alpha)$ and $t(\alpha)$, respectively. A path in Δ is a sequence of arrows $\alpha_1\alpha_2\cdots\alpha_n$ such that $t(\alpha_i) = s(\alpha_{i+1})$ for $i = 1, \ldots, n-1$. The set of all paths of length n is denoted by Δ_n. By adjoining the element $\perp$, we will consider the following set (cf. [11], [12]): $\hat{\Delta} = \{\perp\} \cup \bigcup_{i=0}^{\infty} \Delta_i$. This set is a semigroup with the multiplication defined by

$$\delta \cdot \gamma = \begin{cases} \delta\gamma & \text{if} \quad t(\delta) = s(\gamma), \\ \perp & \text{otherwise}, \end{cases} \quad \delta, \gamma \in \bigcup_{i=0}^{\infty} \Delta_i; \quad \perp \cdot \gamma = \gamma \cdot \perp = \perp, \ \gamma \in \hat{\Delta}.$$

Let K be a commutative ring. Then $K\hat{\Delta}$ is a semigroup algebra and the path algebra $K\Delta$ is isomorphic to $K\hat{\Delta}/(\perp)$. So, $K\Delta$ is a $\hat{\Delta}$-graded algebra with a basis consisting of the paths in Δ. Moreover, $K\Delta$ is $\mathbb{N}$-graded, that is, $K\Delta = \bigoplus_{i=0}^{\infty} K\Delta_i$. In particular, R_Δ^m is $\hat{\Delta}$-graded and $\mathbb{N}$-graded, thus the truncated quiver algebra $A = K\Delta/R_\Delta^m$ is a $\hat{\Delta}$-graded and $\mathbb{N}$-graded algebra.

For an $\mathbb{N}$-graded vector space V, V_+ is defined by $V_+ = \bigoplus_{i \geq 1} V_i$.

Let Δ be a finite quiver. For a path γ, $|\gamma|$ denotes the length of γ. A path γ is said to be a cycle if $|\gamma| \geq 1$ and its source and target coincide. The period of a cycle γ is defined by the smallest integer i such that $\gamma = \delta^j$ $(j \geq 1)$ for a cycle δ of length i, which is denoted by $\operatorname{per}\gamma$. A cycle is said to be a basic cycle if the length of the cycle coincides with its period. It is also called a proper cycle [6]. Denote by Δ_n^{c} (respectively Δ_n^{b}) the set of cycles (respectively basic cycles) of length n. Let $G_n = \langle t_n \rangle$ be the cyclic group of order n and the path $\alpha_1 \cdots \alpha_{n-1}\alpha_n$ a cycle where α_i is an arrow in Δ. Then we define the action of G_n on Δ_n^{c} by $t_n \cdot (\alpha_1 \cdots \alpha_{n-1}\alpha_n) := \alpha_n\alpha_1 \cdots \alpha_{n-1}$, and $\Delta_n^{\mathrm{c}}/G_n$ denotes the set of all G_n-orbits on Δ_n^{c}. Similarly, G_n acts on Δ_n^{b}, and $\Delta_n^{\mathrm{b}}/G_n$ denotes the set of all G_n-orbits on Δ_n^{b}. For $\bar{\gamma} \in \Delta_n^{\mathrm{c}}/G_n$, we define the period $\operatorname{per}\bar{\gamma}$ of $\bar{\gamma}$ by $\operatorname{per}\gamma$. For convenience we use the notation $\Delta_0^{\mathrm{c}}/G_0$ for the set of vertices Δ_0. Throughout this paper, $\alpha_i (i \geq 0)$ denotes an arrow in Δ.

3. The cyclic homology of truncated quiver algebras

In this section, we introduce the projective resolution and the Hochschild homology of truncated quiver algebra in [11], and by means of chain maps which are given by Ames, Cagliero, Tirao, we determine the dimension formula of the cyclic homology of truncated quiver algebra.

Theorem 1 ([11, Theorem 1]). *The following is a projective $\hat{\Delta}$-graded resolution of A as a left A^e-module:*

$$\boldsymbol{P} : \cdots \xrightarrow{d_{i+1}} P_i \xrightarrow{d_i} \cdots \xrightarrow{d_2} P_1 \xrightarrow{d_1} P_0 \xrightarrow{\epsilon} A \longrightarrow 0.$$

Here the modules are defined by $P_i = A \otimes_{K\Delta_0} K\Gamma^{(i)} \otimes_{K\Delta_0} A$, where $\Gamma^{(i)}$ is given by

$$\Gamma^{(i)} = \begin{cases} \Delta_{cm} & \text{if } i = 2c \ (c \geq 0), \\ \Delta_{cm+1} & \text{if } i = 2c+1 \ (c \geq 0), \end{cases}$$

and the differentials are defined by

$$d_{2c}(\alpha\otimes\alpha_1\cdots\alpha_{cm}\otimes\beta)=\sum_{j=0}^{m-1}\alpha\alpha_1\cdots\alpha_j\otimes\alpha_{1+j}\cdots\alpha_{(c-1)m+1+j}\otimes\alpha_{(c-1)m+2+j}\cdots\alpha_{cm}\beta,$$

$$d_{2c+1}(\alpha\otimes\alpha_1\cdots\alpha_{cm+1}\otimes\beta)=\alpha\alpha_1\otimes\alpha_2\cdots\alpha_{cm+1}\otimes\beta-\alpha\otimes\alpha_1\cdots\alpha_{cm}\otimes\alpha_{cm+1}\beta.$$

The augmentation $\varepsilon\colon A\otimes_{K\Delta_0}K\Delta_0\otimes_{K\Delta_0}A\cong A\otimes_{K\Delta_0}A\longrightarrow A$ *is defined by* $\varepsilon(\alpha\otimes\beta)=\alpha\beta$.

Theorem 2 ([11, Theorem 2])**.** *Let K be a commutative ring and A a truncated quiver algebra $K\Delta/R_\Delta^m$ and $q=cm+e$ for $0\le e\le m-1$. Then the degree q part of the pth Hochschild homology $HH_p(A)$ is given by*

$$HH_{p,q}(A)=\begin{cases}K^{a_q} & \text{if } 1\le e\le m-1 \text{ and } 2c\le p\le 2c+1,\\ \bigoplus_{r\mid q}\left(K^{\gcd(m,r)-1}\oplus \mathrm{Ker}\left(\cdot\frac{m}{\gcd(m,r)}:K\longrightarrow K\right)\right)^{b_r} & \\ & \text{if } e=0 \text{ and } 0<2c-1=p,\\ \bigoplus_{r\mid q}\left(K^{\gcd(m,r)-1}\oplus \mathrm{Coker}\left(\cdot\frac{m}{\gcd(m,r)}:K\longrightarrow K\right)\right)^{b_r} & \\ & \text{if } e=0 \text{ and } 0<2c=p,\\ K^{\#\Delta_0} & \text{if } p=q=0,\\ 0 & \text{otherwise.}\end{cases}$$

Here we set $a_q:=\#(\Delta_q^{\mathrm{c}}/G_q)$ *and* $b_r:=\#(\Delta_r^{\mathrm{b}}/G_r)$.

Lemma 3 ([4, Lemma 1.1])**.** *Let Δ be a finite quiver, I an admissible ideal, $K\Delta_0$ the subalgebra of $A=K\Delta/I$ generated by Δ_0 and r the Jacobson radical of A. The following is a projective resolution of A as a left A^e-module:*

$$\boldsymbol{Q}:\cdots\longrightarrow A\otimes_{K\Delta_0}r^{\otimes^i_{K\Delta_0}}\otimes_{K\Delta_0}A\xrightarrow{d_i}A\otimes_{K\Delta_0}r^{\otimes^{i-1}_{K\Delta_0}}\otimes_{K\Delta_0}A\longrightarrow\cdots$$
$$\longrightarrow A\otimes_{K\Delta_0}r\otimes_{K\Delta_0}A\xrightarrow{d_1}A\otimes_{K\Delta_0}A\xrightarrow{d_0}A\longrightarrow 0,$$

where the differentials $d_n(n\ge 0)$ are defined by $d_0(\lambda[\ \ ]\mu)=\lambda\mu$ and $d_i(\lambda[x_1|\cdots|x_i]\mu)=\lambda x_1[x_2|\cdots|x_i]\mu+\sum_{j=1}^{i-1}(-1)^j\lambda[x_1|\cdots|x_jx_{j+1}|\cdots|x_i]\mu+(-1)^i\lambda[x_1|\cdots|x_{i-1}]x_i\mu$ for $i\ge 1$, and we use the bar notation $\lambda[x_1|\cdots|x_i]\mu$ for $\lambda\otimes x_1\otimes x_2\otimes\cdots\otimes x_i\otimes\mu$.

Cibils constructs the following mixed complex.

Theorem 4 ([5], [12])**.** *Let Δ be a finite quiver, K a field, and $A=K\Delta/I$ for I a homogeneous ideal. Define the mixed complex $(C_{K\Delta_0}(A),b,B)$ by $C_{K\Delta_0}(A)_n=A\otimes_{K\Delta_0^e}A_+^{\otimes^n_{K\Delta_0}}$ and*

$$b(x_0[x_1|\cdots|x_n])=x_0x_1[x_2|\cdots|x_n]+\sum_{i=1}^{n-1}(-1)^ix_0[x_1|\cdots|x_ix_{i+1}|\cdots|x_n]$$
$$+(-1)^nx_nx_0[x_1|\cdots|x_{n-1}],$$
$$B(x_0[x_1|\cdots|x_n])=\sum_{i=0}^{n}(-1)^{in}[x_i|\cdots|x_n|x_0|\cdots|x_{i-1}].$$

Then $HH_n(C_{K\Delta_0}(A))=HH_n(A)$ *and* $HC_n(C_{K\Delta_0}(A))=HC_n(A)$.

In particular, if A is a truncated quiver algebra $K\Delta/R_\Delta^m (m \geq 2)$, then the map B in $(C_{K\Delta_0}(A), b, B)$ respects the $\Delta_*^{\mathrm{c}}/G_*$-grading (cf. [12]). Furthermore if we consider the double complex $\mathcal{B}C$ associate to this mixed complex and filter the total complex $\mathrm{Tot}\,\mathcal{B}C$ by the column filtration, then the resulting spectral sequence is $\Delta_*^{\mathrm{c}}/G_*$-graded. Thus $HC_n(A)$ is $\Delta_*^{\mathrm{c}}/G_*$-graded. Moreover, for $\bar{\gamma} \in \Delta_*^{\mathrm{c}}/G_*$ the degree $\bar{\gamma}$ part of the E^1-term of this spectral sequence is $E^1_{p,q,\bar{\gamma}} = HH_{q-p,\bar{\gamma}}(A)$.

On the other hand, Ames, Cagliero and Tirao find the chain maps between the left A^e-projective resolutions $\boldsymbol{P}$ and $\boldsymbol{Q}$ of a truncated quiver algebra A over an arbitrary field as follows:

Proposition 5 ([1])**.** *Define the map* $\iota : \boldsymbol{P} \longrightarrow \boldsymbol{Q}$ *as follows*:

$$\iota_0(\alpha \otimes \beta) = \alpha[\quad]\beta,\ \iota_1(\alpha \otimes \alpha_1 \otimes \beta) = \alpha[\alpha_1]\beta,$$

$$\begin{aligned}&\iota_{2c}(\alpha \otimes \alpha_1 \cdots \alpha_{cm} \otimes \beta)\\ &= \sum_{0 \leq j_1, \ldots, j_c \leq m-2} \alpha[\alpha_1 \cdots \alpha_{1+j_1}|\alpha_{2+j_1}|\alpha_{3+j_1} \cdots \alpha_{3+j_1+j_2}|\alpha_{4+j_1+j_2}| \cdots \\ &\qquad |\alpha_{2c-1+j_1+\cdots+j_{c-1}} \cdots \alpha_{2c-1+j_1+\cdots+j_c}|\alpha_{2c+j_1+\cdots+j_c}]\alpha_{2c+1+j_1+\cdots+j_c} \cdots \alpha_{cm}\beta,\end{aligned}$$

$$\begin{aligned}&\iota_{2c+1}(\alpha \otimes \alpha_1 \cdots \alpha_{cm+1} \otimes \beta)\\ &= \sum_{0 \leq j_1, \ldots, j_c \leq m-2} \alpha[\alpha_1|\alpha_2 \cdots \alpha_{2+j_1}|\alpha_{3+j_1}|\alpha_{4+j_1} \cdots \alpha_{4+j_1+j_2}|\alpha_{5+j_1+j_2}| \cdots \\ &\qquad |\alpha_{2c+j_1+\cdots+j_{c-1}} \cdots \alpha_{2c+j_1+\cdots+j_c}|\alpha_{2c+1+j_1+\cdots+j_c}]\alpha_{2c+2+j_1+\cdots+j_c} \cdots \alpha_{cm+1}\beta.\end{aligned}$$

Then, ι *is a chain map.*

Proposition 6 ([1])**.** *Let* m_i *be a positive integer for any* $i \geq 1$*. Suppose that* x_i *is the path* $\alpha_{m_1+\cdots+m_{i-1}+1} \cdots \alpha_{m_1+\cdots+m_i}$ *of length* m_i*. Define the map* $\pi : \boldsymbol{Q} \longrightarrow \boldsymbol{P}$ *as follows*:

$$\pi_0(\alpha[\quad]\beta) = \alpha \otimes \beta,$$

$$\pi_1(\alpha[x_1]\beta) = \sum_{j=1}^{m_1} \alpha\alpha_1 \cdots \alpha_{j-1} \otimes \alpha_j \otimes \alpha_{j+1} \cdots \alpha_{m_1}\beta,$$

$$\pi_{2c}(\alpha[x_1|x_2|\cdots|x_{2c}]\beta) = \begin{cases} \alpha \otimes \alpha_1 \cdots \alpha_{cm} \otimes \alpha_{cm+1} \cdots \alpha_{m_1+\cdots+m_{2c}}\beta \\ \qquad\qquad \text{if} \quad m_{2i-1} + m_{2i} \geq m\ (1 \leq i \leq c), \\ 0 \qquad \text{otherwise}, \end{cases}$$

$$\pi_{2c+1}(\alpha[x_1|x_2|\cdots|x_{2c+1}]\beta) = \begin{cases} \sum_{j=1}^{m_1} \alpha\alpha_1 \cdots \alpha_{j-1} \otimes \alpha_j \cdots \alpha_{j+cm} \otimes \\ \qquad\qquad \alpha_{j+cm+1} \cdots \alpha_{m_1+\cdots+m_{2c+1}}\beta \\ \qquad \text{if} \quad m_{2i} + m_{2i+1} \geq m\ (1 \leq i \leq c), \\ 0 \qquad \text{otherwise}. \end{cases}$$

Then, π *is a chain map and* $\pi\iota = \mathrm{id}_{\boldsymbol{P}}$*.*

By investigating the basis of the Hochschild homology and finding the chain maps between the projective resolutions $\boldsymbol{P}$ and $\boldsymbol{Q}$, we are able to compute $B : HH_{p,\bar{\gamma}}(A) \longrightarrow HH_{p+1,\bar{\gamma}}(A)$ induced by the differential of the Cibils' mixed complex. Moreover, for $\bar{\gamma} \in \Delta_t^{\mathrm{c}}/G_t$ we are able to determine the degree $\bar{\gamma}$ part of the E^2-term of the spectral sequence associated with the Cibils' mixed complex. Therefore we have the following result.

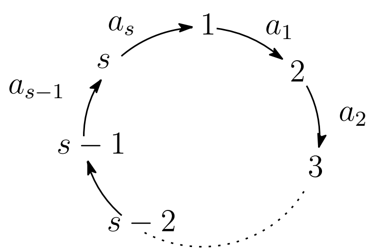
Theorem 7 ([8, Theorem 5.1])**.** *Suppose that $m \geq 2$ and $A = K\Delta/R_\Delta^m$. Then the dimension formula of the cyclic homology of A is given by, for $c \geq 0$,*

$$\dim_K HC_{2c}(A) = \#\Delta_0 + \sum_{e=1}^{m-1} a_{cm+e} + \sum_{c'=0}^{c-1}\sum_{e=1}^{m-1} \sum_{\substack{r>0 \\ \text{s.t. } r\zeta | c'm+e}} b_r + \sum_{c'=1}^{c} \sum_{\substack{r>0 \\ \text{s.t. } r|c'm, \\ \gcd(m,r)\zeta|m}} b_r + \sum_{c'=1}^{c} \sum_{\substack{r>0 \\ \text{s.t.} r\zeta|\gcd(m,r)c'}} (\gcd(m,r)-1)b_r,$$

$$\dim_K HC_{2c+1}(A) = \sum_{\substack{r>0 \\ \text{s.t. } r|(c+1)m}} (\gcd(m,r)-1)b_r + \sum_{c'=0}^{c}\sum_{e=1}^{m-1} \sum_{\substack{r>0 \\ \text{s.t. } r\zeta|c'm+e}} b_r + \sum_{c'=1}^{c+1} \sum_{\substack{r>0 \\ \text{s.t. } r|c'm, \\ \gcd(m,r)\zeta|m}} b_r + \sum_{c'=1}^{c} \sum_{\substack{r>0 \\ \text{s.t. } r\zeta|\gcd(m,r)c'}} (\gcd(m,r)-1)b_r.$$

Remark 8. If $\zeta = 0$, then the above result coincides with the result of Taillefer in [13].

Example 9 ([8, Example 5.3])**.** Let K be a field of characteristic ζ and Δ the following quiver:



Suppose $m \geq 2$ and $A = K\Delta/R_\Delta^m$, which is called a truncated cycle algebra in [2]. Since

$$a_r = \begin{cases} 1 & \text{if } \ s|r, \\ 0 & \text{otherwise,} \end{cases} \qquad b_r = \begin{cases} 1 & \text{if } \ s=r, \\ 0 & \text{otherwise,} \end{cases}$$

we have, for $c \geq 0$,

$$\dim_K HC_{2c}(A) = s + \left[\frac{(c+1)m-1}{s}\right] - \left[\frac{cm}{s}\right] + \sum_{c'=0}^{c-1}\left(\left[\frac{(c'+1)m-1}{s\zeta}\right] - \left[\frac{c'm}{s\zeta}\right]\right) + \left(\left[\frac{m}{\gcd(m,s)\zeta}\right] - \left[\frac{m-1}{\gcd(m,s)\zeta}\right]\right)\sum_{c'=1}^{c}\left(\left[\frac{c'm}{s}\right] - \left[\frac{c'm-1}{s}\right]\right) + (\gcd(m,s)-1)\left[\frac{\gcd(m,s)c}{s\zeta}\right],$$

$$\dim_K HC_{2c+1}(A) = (\gcd(m,s)-1)\left(\left[\frac{(c+1)m}{s}\right]-\left[\frac{(c+1)m-1}{s}\right]+\left[\frac{\gcd(m,s)c}{s\zeta}\right]\right)$$
$$+\left(\left[\frac{m}{\gcd(m,s)\zeta}\right]-\left[\frac{m-1}{\gcd(m,s)\zeta}\right]\right)\sum_{c'=1}^{c+1}\left(\left[\frac{c'm}{s}\right]-\left[\frac{c'm-1}{s}\right]\right)$$
$$+\sum_{c'=0}^{c}\left(\left[\frac{(c'+1)m-1}{s\zeta}\right]-\left[\frac{c'm}{s\zeta}\right]\right).$$

4. The m-truncated cycles version of the "no loops conjecture"

In this section, we introduce the result that if an algebra $K\Delta/I$ with $I \subset R_\Delta^m$ has an m-truncated cycle (see Definition 10), then the algebra has infinite Hochschild homology dimension. Moreover, we show that the algebra $K\Delta/I$ satisfies the m-truncated cycles version of the "no loops conjecture".

If $I \subset R_\Delta^2$ is an ideal in the path algebra $K\Delta$, then a finite sequence $\alpha_1, \ldots, \alpha_u$ of arrows which satisfies the equations $t(\alpha_i) = s(\alpha_{i+1})$ $(i = 1, \ldots, u-1)$ and $t(\alpha_u) = s(\alpha_1)$ is called *a cycle* in $K\Delta/I$ in [3].

Definition 10 ([3]). A cycle $\alpha_1, \ldots, \alpha_u$ in $K\Delta/I$ is *m-truncated* for an integer $m \geq 2$ if

$$\alpha_i \cdots \alpha_{i+m-1} = 0 \quad \text{and} \quad \alpha_i \cdots \alpha_{i+m-2} \neq 0 \quad \text{in } K\Delta/I$$

for all i, where the indices are modulo u.

In order to describe the result which is used that the m-truncated cycles of the no loops conjecture we recall that the Hochschild homology dimension of the algebra A is defined by $\mathrm{HHdim}\, A := \sup\{n \in \mathbb{Z} \mid HH_n(A) \neq 0\}$.

Theorem 11 ([9]). *Let K be a field, Δ a finite quiver and $I \subset K\Delta$ an ideal contained in R_Δ^m. Suppose that $K\Delta/I$ contains an m-truncated cycle $\alpha_1, \ldots, \alpha_u$. Then for every $n \geq 1$ with $un \equiv 0 \pmod m$, the element*

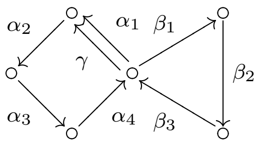
$$\alpha_{(c-1)m+2} \cdots \alpha_{cm} \otimes \alpha_1 \otimes \alpha_2 \cdots \alpha_m \otimes \alpha_{m+1}$$
$$\otimes \alpha_{m+2} \cdots \alpha_{2m} \otimes \alpha_{2m+1} \otimes \cdots \otimes \alpha_{(c-2)m+2} \cdots \alpha_{(c-1)m} \otimes \alpha_{(c-1)m+1},$$

where $c = un/m$, represents a nonzero element in $HH_{2c-1}(K\Delta/I)$. In particular, the Hochschild homology dimension $\mathrm{HHdim}\,(K\Delta/I) = \infty$.

For a basic and connected finite dimensional K-algebra A, in the case the ground field K is an algebraically closed field, it is well known that the projective dimension of A as a left A^e-module is equal to global dimension $\mathrm{gl.dim}\, A$ of A (cf. [7]). Hence, $\mathrm{HHdim}\, A \leq \mathrm{gl.dim}\, A$. Therefore, by the above theorem, we have the following result which generalizes [3, Collorary 3.3] in the case the ground field is an algebraically closed field.

Corollary 12 ([9]). *Let K be an algebraically closed field, Δ a finite quiver and I an admissible ideal in $K\Delta$ with $I \subset R_\Delta^m$. If the algebra $K\Delta/I$ has finite global dimension, then it contains no m-truncated cycles.*

Example 13 ([9])**.** Let A be an algebra given by the quiver



with relations:

$$\alpha_i\alpha_{i+1}\alpha_{i+2} = \beta_1\beta_2\beta_3 = 0,\ \beta_2\beta_3\alpha_1 = \beta_2\beta_3\gamma,$$

where the indices of α_i are modulo 4 ($1 \leq i \leq 4$). Then A has the 3-truncated cycle $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. By the Theorem 11, for every $n \geq 1$ with $n \equiv 0 \pmod 3$, the element $(\alpha_3\alpha_4 \otimes \alpha_1 \otimes \alpha_2\alpha_3 \otimes \alpha_4 \otimes \alpha_1\alpha_2 \otimes \alpha_3 \otimes \alpha_4\alpha_1 \otimes \alpha_2)^{\otimes(n/3)}$ is a nonzero element in $HH_{(8n/3)-1}(A)$. So we have $\operatorname{HHdim} A = \infty$. Therefore, the global dimension of A is infinite.

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Department of Mathematics
Tokyo University of Science
1-3 Kagurazaka, Shinjuku-ku, Tokyo 162-8601 JAPAN
E-mail address: 1112701@ed.tus.ac.jp