Sage Quick Reference: Abstract Algebra
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latest version: `http://wiki.sagemath.org/quickref`

Based on work by P. Jipsen, W. Stein, R. Beezer

全般的なこと　Basic Help

com⟨tab⟩　*command* が補完される
`a.`⟨tab⟩　オブジェクト a のメソッドの一覧を表示
command?　簡単な情報と例を表示
command??　ソースコードを表示
`_`　直前の出力
`www.sagemath.org/doc/reference`　リファレンス
`www.sagemath.org/doc/tutorial`　チュートリアル
`load foo.sage`　ファイル foo.sage からコマンドをロード
`attach foo.sage`　foo.sage が更新されたら自動的にロード

ORGINAL TEXT

com⟨tab⟩　complete *command*
`a.`⟨tab⟩　all methods for object a
command?　for summary and examples
command??　for complete source code
`_`　underscore gives the previous output
`www.sagemath.org/doc/reference`　online reference
`www.sagemath.org/doc/tutorial`　online tutorial
`load foo.sage`　load commands from the file foo.sage
`attach foo.sage`　loads changes to foo.sage automatically

リスト　Lists

`L = [2,17,3,17]`　リスト
`L[i]`　L の i 番目の要素
　Note: 0 番目の要素からリストは始まっている
`L.append(x)`　L の最後の要素として x を加える
`L.remove(x)`　L から x を除く
`L[i:j]`　L の i 番目から $(j-1)$ 番目までの要素
`range(a)`　0 から $a-1$ までの整数のリスト
`range(a,b)`　a から $b-1$ までの整数のリスト
`[a..b]`　a から b までの整数のリスト
`range(a,b,c)`　$a, a+c, a+2c, \ldots$ のうち b を超えないもの
`len(L)`　length of L
`M = [i^2 for i in range(13)]`
　0 から 12 までの整数の平方からなるリスト
`N = [i^2 for i in range(13) if is_prime(i)]`
　0 から 12 までの整数のうち素数であるものからなるリスト
`M + N`　M と N をつなげたリスト.
`sorted(L)`　L の要素をソートしたリスト (L は変更されない)
`L.sort()`　L をソート (L が変更される)
`set(L)`　重複する要素はなく, 順序を気にしないリスト (集合)

ORGINAL TEXT

`L = [2,17,3,17]`　an ordered list
`L[i]`　the ith element of L
　Note: lists begin with the 0th element
`L.append(x)`　adds x to L
`L.remove(x)`　removes x from L
`L[i:j]`　the i-th through $(j-1)$-th element of L
`range(a)`　list of integers from 0 to $a-1$
`range(a,b)`　list of integers from a to $b-1$
`[a..b]`　list of integers from a to b
`range(a,b,c)`
　every c-th integer starting at a and less than b
`len(L)`　length of L
`M = [i^2 for i in range(13)]`
　list of squares of integers 0 through 12
`N = [i^2 for i in range(13) if is_prime(i)]`
　list of squares of prime integers between 0 and 12
`M + N`　the concatenation of lists M and N
`sorted(L)`　a sorted version of L (L is not changed)
`L.sort()`　sorts L (L is changed)
`set(L)`　an unordered list of unique elements

プログラミングの例　Programming Examples

整数 $0, \ldots, 14$ の平方を表示:

```
for i in range(15):
   print i^2
```

0 以上 14 以下の整数のうち 15 と互いに素なものの平方を表示:

```
for i in range(13):
   if gcd(i,15)==1:
     print i^2
```

ORGINAL TEXT

Print the squares of the integers $0, \ldots, 14$:

```
for i in range(15):
   print i^2
```

Print the squares of those integers in $\{0, \ldots, 14\}$ that are relatively prime to 15:

```
for i in range(13):
   if gcd(i,15)==1:
     print i^2
```

基本的な操作　Preliminary Operations

`a = 3; b = 14`
`gcd(a,b)`　a, b の最大公約数
`xgcd(a,b)`　$d = sa + tb$ と $d = \gcd(a,b)$ を満たす三つ組 (d,s,t)
`next_prime(a)`　(a より大きな) a の次の素数
`previous_prime(a)`　(a よりも小さな) a の直前の素数
`prime_range(a,b)`　a 以上 b 未満である素数達
`is_prime(a)`　a は素数か?
`b % a`　b を a で割った余り
`a.divides(b)`　a は b を割り切るか?

ORGINAL TEXT

`a = 3; b = 14`
`gcd(a,b)`　greatest common divisor a, b
`xgcd(a,b)`　triple (d,s,t) where $d = sa + tb$ and $d = \gcd(a,b)$
`next_prime(a)`　next prime after a
`previous_prime(a)`　prime before a
`prime_range(a,b)`　primes p such that $a \leq p < b$
`is_prime(a)`　is a prime?
`b % a`　the remainder of b upon division by a
`a.divides(b)`　does a divide b?

群の構成　Group Constructions

Note: 置換群の積は左から右.
`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
　$(1,2,3)(4,5)$ と $(3,4)$ を生成元とする置換群
`G = PermutationGroup(["(1,2,3)(4,5)","(3,4)"])`
　置換群を定義する別の方法
`S = SymmetricGroup(4)`　対称群, S_4
`A = AlternatingGroup(4)`　交代群, A_4
`D = DihedralGroup(5)`　位数 10 の二面体群
`Ab = AbelianGroup([0,2,6])`　$\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_6$
`Ab.0, Ab.1, Ab.2`　Ab の生成元達
`a,b,c = Ab.gens()`
　`a = Ab.0; b = Ab.1; c = Ab.2` を短く書く方法
`C = CyclicPermutationGroup(5)`
`Integers(8)`　$\mathbb{Z}_8$
`GL(3,QQ)`　3×3 行列からなる一般線形群
`m = matrix(QQ,[[1,2],[3,4]])`
`n = matrix(QQ,[[0,1],[1,0]])`
`MatrixGroup([m,n])`　m と n を生成元とする行列群 (無限群)
`u = S([(1,2),(3,4)]); v = S((2,3,4))`　S の元
`S.subgroup([u,v])`　u と v で生成される S の部分群
`S.quotient(A)`　剰余群 S/A
`A.cartesian_product(D)`　直積群 $A \times D$
`A.intersection(D)`　A と D の共通部分
`D.conjugate(v)`　$v^{-1}Dv$
`S.sylow_subgroup(2)`　S の 2-Sylow 部分群
`D.center()`　D の中心
`S.centralizer(u)`　S での u の中心化群
`S.centralizer(D)`　S での D の中心化群
`S.normalizer(u)`　S での u の正規化群
`S.normalizer(D)`　S での D の正規化群
`S.stabilizer(3)`　3 を固定する S の部分群

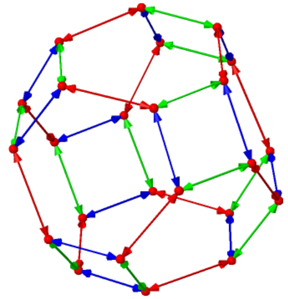
ORGINAL TEXT

Note: Permutation multiplication is left-to-right.
`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
　perm. group with generators $(1,2,3)(4,5)$ and $(3,4)$
`G = PermutationGroup(["(1,2,3)(4,5)","(3,4)"])`
　alternative syntax for defining a permutation group
`S = SymmetricGroup(4)`　the symmetric group, S_4

`A = AlternatingGroup(4)` alternating group, A_4
`D = DihedralGroup(5)` dihedral group of order 10
`Ab = AbelianGroup([0,2,6])` the group $\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_6$
`Ab.0, Ab.1, Ab.2` the generators of `Ab`
`a,b,c = Ab.gens()`
shorthand for `a = Ab.0; b = Ab.1; c = Ab.2`
`C = CyclicPermutationGroup(5)`
`Integers(8)` the group $\mathbb{Z}_8$
`GL(3,QQ)` general linear group of 3×3 matrices
`m = matrix(QQ,[[1,2],[3,4]])`
`n = matrix(QQ,[[0,1],[1,0]])`
`MatrixGroup([m,n])`
the (infinite) matrix group with generators `m` and `n`
`u = S([(1,2),(3,4)]); v = S((2,3,4))` elements of S
`S.subgroup([u,v])` the subgroup of S generated by u, v
`S.quotient(A)` the quotient group S/A
`A.cartesian_product(D)` the group $A \times D$
`A.intersection(D)` the intersection of groups A and D
`D.conjugate(v)` the group $v^{-1}Dv$
`S.sylow_subgroup(2)` a Sylow 2-subgroup of S
`D.center()` the center of D
`S.centralizer(u)` the centralizer of u in S
`S.centralizer(D)` the centralizer of D in S
`S.normalizer(u)` the normalizer of u in S
`S.normalizer(D)` the normalizer of D in S
`S.stabilizer(3)` subgroup of S fixing 3

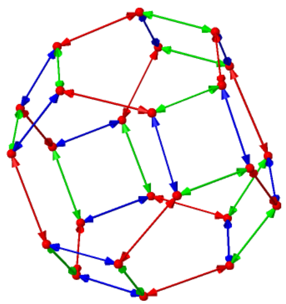
群の操作 Group Operations

`S = SymmetricGroup(4); A = AlternatingGroup(4)`
`S.order()` S の元の個数
`S.gens()` S の生成系
`S.list()` S の元達
`S.random_element()` S の元をランダムに出力
`u*v` S の u と v の積
`v^(-1)*u^3*v` S の元 $v^{-1}u^3v$
`u.order()` u の位数
`S.subgroups()` S の部分群達
`S.normal_subgroups()` S の正規部分群達
`A.cayley_table()` A の乗積表
`u in S` u は S の元か?
`u.word_problem(S.gens())` S の生成元の積として u を書く
`A.is_abelian()` A は可換か?
`A.is_cyclic()` A は巡回群か?
`A.is_simple()` A は単純群か?
`A.is_transitive()` A は可移置換群か?
`A.is_subgroup(S)` A は S の部分群か?
`A.is_normal(S)` A は S の正規部分群か?
`S.cosets(A)` S での A の右剰余類
`S.cosets(A,'left')` S での A の左剰余類
`g = S.cayley_graph()` S の Cayley graph
`g.show3d(color_by_label=True, edge_size=0.01, vertex_size=0.03)`



ORGINAL TEXT

`S = SymmetricGroup(4); A = AlternatingGroup(4)`
`S.order()` the number of elements of S
`S.gens()` generators of S
`S.list()` the elements of S
`S.random_element()` a random element of S
`u*v` the product of elements u and v of S
`v^(-1)*u^3*v` the element $v^{-1}u^3v$ of S
`u.order()` the order of u
`S.subgroups()` the subgroups of S
`S.normal_subgroups()` the normal subgroups of S
`A.cayley_table()` the multiplication table for A
`u in S` is u an element of S?
`u.word_problem(S.gens())`
write u as a product of the generators of S
`A.is_abelian()` is A abelian?
`A.is_cyclic()` is A cyclic?
`A.is_simple()` is A simple?
`A.is_transitive()` is A transitive?
`A.is_subgroup(S)` is A a subgroup of S?
`A.is_normal(S)` is A a normal subgroup of S?
`S.cosets(A)` the right cosets of A in S
`S.cosets(A,'left')` the left cosets of A in S
`g = S.cayley_graph()` Cayley graph of S
`g.show3d(color_by_label=True, edge_size=0.01, vertex_size=0.03)`



環と体の構成 Ring and Field Constructions

`ZZ` 整数からなる整域, $\mathbb{Z}$
`Integers(7)` 7 を法とした整数の環, $\mathbb{Z}_7$
`QQ` 有理数体, $\mathbb{Q}$
`RR` 実数からなる体, $\mathbb{R}$
`CC` 複素数体, $\mathbb{C}$
`RDF` 浮動小数点 (double) を使った実数の体, (近似)
`CDF` 浮動小数点 (double) を使った実数の複素数体, (近似)
`RR` 53 ビット実数による体, (近似), `RDF` とは別物
`RealField(400)` 400 ビット実数による体, (近似)
`ComplexField(400)` 複素数も同様
`ZZ[I]` ガウス整数環
`QuadraticField(7)` 二次体, $\mathbb{Q}(\sqrt{7})$
`CyclotomicField(7)` $\mathbb{Q}$ と x^7-1 の根を含む最小の体
`AA, QQbar` 代数的数のなす体, $\overline{\mathbb{Q}}$
`FiniteField(7)` $\mathbb{Z}_7$
`F.<a> = FiniteField(7^3)`
7^3 個の元からなる有限体, a は生成元, $GF(7^3)$
`SR` シンボリックな数式のなす環
`M.<a>=QQ[sqrt(3)]` $M = \mathbb{Q}[\sqrt{3}]$, $a = \sqrt{3}$.
`A.<a,b>=QQ[sqrt(3),sqrt(5)]`
$M = \mathbb{Q}[\sqrt{3}, \sqrt{5}]$, $a = \sqrt{3}$, $b = \sqrt{5}$.
`z = polygen(QQ,'z'); K = NumberField(x^2 - 2,'s')`
$x^2 - 2$ の根 s を含む数体
`s = K.0` `s` を `K` の生成元とする
`D = ZZ[sqrt(3)]; D.fraction_field()` 整域 `D` の商体.

ORIGINAL TEXT

`ZZ` integral domain of integers, $\mathbb{Z}$
`Integers(7)` ring of integers mod 7, $\mathbb{Z}_7$
`QQ` field of rational numbers, $\mathbb{Q}$
`RR` field of real numbers, $\mathbb{R}$
`CC` field of complex numbers, $\mathbb{C}$
`RDF` real double field, inexact
`CDF` complex double field, inexact
`RR` 53-bit reals, inexact, not same as `RDF`
`RealField(400)` 400-bit reals, inexact
`ComplexField(400)` complexes, too
`ZZ[I]` the ring of Gaussian integers
`QuadraticField(7)` the quadratic field, $\mathbb{Q}(\sqrt{7})$
`CyclotomicField(7)`
smallest field containing $\mathbb{Q}$ and the zeros of $x^7 - 1$
`AA, QQbar` field of algebraic numbers, $\overline{\mathbb{Q}}$
`FiniteField(7)` the field $\mathbb{Z}_7$
`F.<a> = FiniteField(7^3)`
finite field in a of size 7^3, $GF(7^3)$
`SR` ring of symbolic expressions
`M.<a>=QQ[sqrt(3)]` the field $\mathbb{Q}[\sqrt{3}]$, with $a = \sqrt{3}$.
`A.<a,b>=QQ[sqrt(3),sqrt(5)]`
the field $\mathbb{Q}[\sqrt{3}, \sqrt{5}]$ with $a = \sqrt{3}$ and $b = \sqrt{5}$.
`z = polygen(QQ,'z'); K = NumberField(x^2 - 2,'s')`
the number field in s with defining polynomial $x^2 - 2$
`s = K.0` set `s` equal to the generator of `K`
`D = ZZ[sqrt(3)]`
`D.fraction_field()`
field of fractions for the integral domain `D`

環の操作 Ring Operations

Note: 扱う環によって異なる場合がある

`A = ZZ[I]; D = ZZ[sqrt(3)]`

`A.is_ring()`　A は環か?

`A.is_field()`　A は体か?

`A.is_commutative()`　A は可換か?

`A.is_integral_domain()`　A は整域か?

`A.is_finite()`　A は有限か?

`A.is_subring(D)`　A は D の部分環か?

`A.order()`　A の元の個数

`A.characteristic()`　A の標数

`A.zero()`　A の加法単位元 (0)

`A.one()`　A の積単位元 (1)

`A.is_exact()`
　A の中に浮動小数点を使った元があれば元 `False`.

`a, b = D.gens(); r = a + b`

`r.parent()`　r を含んでいる環 (この場合は, `D`)

`r.is_unit()`　r は単元か?

. . . . ORGINAL TEXT

Note: Operations may depend on the ring

`A = ZZ[I]; D = ZZ[sqrt(3)]`　some rings

`A.is_ring()`　is A a ring?

`A.is_field()`　is A a field?

`A.is_commutative()`　is A commutative?

`A.is_integral_domain()`　is A an integral domain?

`A.is_finite()`　is A is finite?

`A.is_subring(D)`　is A a subring of D?

`A.order()`　the number of elements of A

`A.characteristic()`　the characteristic of A

`A.zero()`　the additive identity of A

`A.one()`　the multiplicative identity of A

`A.is_exact()`
　`False` if A uses a floating point representation

`a, b = D.gens(); r = a + b`

`r.parent()`　the parent ring of r (in this case, `D`)

`r.is_unit()`　is r a unit?

多項式　Polynomials

`R.<x> = ZZ[ ]`　`R` は多項式環 $\mathbb{Z}[x]$.

`R.<x> = QQ[ ]`　`R` は多項式環 $\mathbb{Q}[x]$

　$\mathbb{Q}[x]$ の別の定義法:
　　`R = PolynomialRing(QQ,'x')`
　　`R = QQ['x']`

`S.<z> = Integers(8)[ ]`　`S` は多項式環 $\mathbb{Z}_8[z]$

`S.<s, t> = QQ[ ]`　`S` は多項式環 g $\mathbb{Q}[s,t]$

`p = 4*x^3 + 8*x^2 - 20*x - 24`　`R` $(=\mathbb{Q}[x])$ の元

`p.is_irreducible()`　p は $\mathbb{Q}[x]$ 上既約か?

`q = p.factor()`　p の因数分解

`q.expand()`　`q` を展開

`p.subs(x=3)`　p の $x=3$ での値

`R.ideal(p)`　p で生成される R のイデアル

`R.cyclotomic_polynomial(7)`
　円分多項式 $x^6+x^5+x^4+x^3+x^2+x+1$

`q = x^2-1`

`p.divides(q)`　p は q を割り切るか?

`p.quo_rem(q)`　p を q で割って得られる商と余り

`gcd(p, q)`　p と q の最大公約元

`p.xgcd(q)`　p と q の拡張最大公約元.

`I = S.ideal([s*t+2,s^3-t^2])`
　S $(=\mathbb{Q}[s,t])$ のイデアル $(st+2, s^3-t^2)$

`S.quotient(I)`　剰余環, S/I

. . . . ORGINAL TEXT

`R.<x> = ZZ[ ]`　`R` is the polynomial ring $\mathbb{Z}[x]$

`R.<x> = QQ[ ]`　`R` is the polynomial ring $\mathbb{Q}[x]$

　alternative syntax for defining the polynomial ring $\mathbb{Q}[x]$:
　　`R = PolynomialRing(QQ,'x')`
　　`R = QQ['x']`

`S.<z> = Integers(8)[ ]`　`S` is the polynomial ring $\mathbb{Z}_8[z]$

`S.<s, t> = QQ[ ]`　`S` is the polynomial ring $\mathbb{Q}[s,t]$

`p = 4*x^3 + 8*x^2 - 20*x - 24`
　a polynomial in `R` $(=\mathbb{Q}[x])$

`p.is_irreducible()`　is p irreducible over $\mathbb{Q}[x]$?

`q = p.factor()`　factor p

`q.expand()`　expand `q`

`p.subs(x=3)`　evaluates p at $x=3$

`R.ideal(p)`　the ideal in R generated by p

`R.cyclotomic_polynomial(7)`
　the cyclotomic polynomial $x^6+x^5+x^4+x^3+x^2+x+1$

`q = x^2-1`

`p.divides(q)`　does p divide q?

`p.quo_rem(q)`
　the quotient and remainder of p upon division by q

`gcd(p, q)`　the greatest common divisor of p and q

`p.xgcd(q)`　the extended gcd of p and q

`I = S.ideal([s*t+2,s^3-t^2])`
　the ideal $(st+2, s^3-t^2)$ in S $(=\mathbb{Q}[s,t])$

`S.quotient(I)`　the quotient ring, S/I

体の操作　Field Operations

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`

`C.<c> = A.absolute_field()`
　"flattens" a relative field extension

`A.relative_degree()`
　the degree of the relative extension field

`A.absolute_degree()`
　the degree of the absolute extension

`r = a + b; r.minpoly()`　r の最小多項式.

`C.is_galois()`　C は $\mathbb{Q}$ の Galois 拡大か?

. . . . ORGINAL TEXT

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`

`C.<c> = A.absolute_field()`
　"flattens" a relative field extension

`A.relative_degree()`
　the degree of the relative extension field

`A.absolute_degree()`
　the degree of the absolute extension

`r = a + b; r.minpoly()`
　the minimal polynomial of the field element r

`C.is_galois()`　is C a Galois extension of Q?